

## Volumes of Solids Using Cylindrical Shells

cylindrical shell - a solid enclosed by two concentric right circular cylinders

$$\begin{aligned}
 V &= \text{area of cross section} \cdot \text{height} \\
 &= (\pi r_2^2 - \pi r_1^2) h \\
 &= \pi (r_2^2 - r_1^2) h \\
 &= \pi (r_2 - r_1)(r_2 + r_1) \cdot h \\
 &= 2\pi \left[ \frac{1}{2} (r_1 + r_2) \right] \cdot h \cdot (r_2 - r_1)
 \end{aligned}$$



where

$\frac{1}{2} (r_1 + r_2)$  is avg radius

$r_2 - r_1$  is thickness

$$\begin{aligned}
 \therefore V &= 2\pi \cdot \text{avg radius} \cdot h \cdot \text{thickness} \\
 &= \int_a^b 2\pi x f(x) dx
 \end{aligned}$$

$y = \sqrt{x}, x=1, x=4, x\text{-axis}$

$$\begin{aligned}
 1) \quad V &= \int_1^4 2\pi x (\sqrt{x}) dx \\
 &= 2\pi \int_1^4 x^{3/2} dx \\
 &= 2\pi \left[ \frac{2}{5} x^{5/2} \right]_1^4 \\
 &= \frac{124\pi}{5}
 \end{aligned}$$