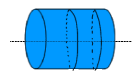
Volumes of Revolution

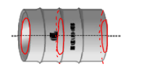
Using Washers and Disks

Recall: $V = A \cdot h$

$= (\text{area of cross section} \times \text{height})$



$V = \pi r^2 h$



$V = (\pi r_2^2 - \pi r_1^2)\, h$

Volume Formula

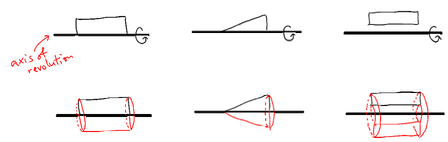
Let S be a solid bounded by two parallel planes $\perp$ to the x-axis at $x = a$ and $x = b$.
If for each x in $[a, b]$, the cross-sectional area of S $\perp$ to the x-axis is $A(x)$, then the volume of the solid is

$$V = \int_a^b A(x)\,dx \qquad \text{provided } A(x) \text{ is integrable}$$

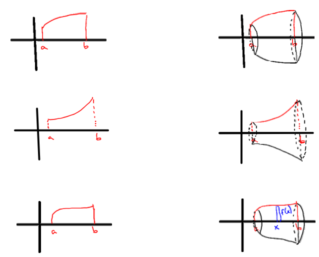
for cross sections $\perp$ to the y-axis

$$V = \int_c^d A(y)\,dy$$

A solid of revolution is a solid that is generated by revolving a plane region about a line that lies in the same plane as the region
(this line is called the axis of revolution)



ex. Sketch the solids generated by revolving the following about the x-axis



$r = f(x)$

$A = \pi r^2$
$= \pi (f(x))^2$

$\therefore$ Volume of the solid is

$$V = \int_a^b \pi\,[f(x)]^2\,dx$$

ex.1 Find the volume of the solid that is obtained when the region under the curve $y = \sqrt{x}$ over the interval $[1, 4]$ is revolved around the x-axis.

$$\begin{aligned}
V &= \int_a^b \pi\,[f(x)]^2\,dx \\
&= \int_1^4 \pi (\sqrt{x})^2\,dx \\
&= \pi \int_1^4 x\,dx \\
&= \pi \left[\frac{x^2}{2}\right]_1^4 \\
&= \pi \left[8 - \frac{1}{2}\right] \\
&= \frac{15\pi}{2}\,u^3
\end{aligned}$$

ex.2 Determine the volume of the solid obtained by rotating the region bounded by $y = x^2 - 4x + 5$, $x = 1$, $x = 4$ and the x-axis about the x-axis

$$\begin{aligned}
V &= \int_a^b \pi\,[f(x)]^2\,dx \\
&= \pi \int_1^4 (x^2 - 4x + 5)^2\,dx \\
&= \pi \int_1^4 (x^4 - 8x^3 + 26x^2 - 40x + 25)\,dx \\
&= \pi \left[\frac{x^5}{5} - 2x^4 + \frac{26x^3}{3} - 20x^2 + 25x\right]_1^4 \\
&= \frac{78\pi}{5}\,u^3
\end{aligned}$$

Assign
Determine the volumes bounded by the given curves around the x-axis

1) $y = \sqrt{3 - x}$, $x = -1$, $x = 3$
2) $y = x^2$, $x = 0$, $x = 2$
3) $y = \sqrt{9 - x^2}$, $y = 0$
4) $y = x - x^2$, $y = 0$

1) 8π
2) $\frac{32\pi}{5}$
3) 36π
4) $\frac{\pi}{30}$