## Trigonometric Functions

## January 2014

Question 33
a) 1 mark
b) 2 marks

The following graph represents tidal levels in the Bay of Fundy over a 25-hour period.


## Solution

a) 6 metres

1 mark

1 mark for period

$$
=12.5 \text { hours }
$$

The period represents the time to complete one cycle of tidal levels in the Bay of Fundy.

1 mark for explanation
2 marks

Question 43
Sketch a graph of at least one period of the function $y=5 \sin [\pi(x+1)]$.
Clearly indicate the $x$-intercepts.

## Solution

$b=\pi$
$\therefore$ period $=\frac{2 \pi}{\pi}=2$


> 1 mark for amplitude
> 1 mark for horizontal shift
> 1 mark for period
> 1 mark for clearly indicating at least two $x$-intercepts consistent with graph

4 marks

## Question 9

1 mark

Sketch the angle of 5 radians in standard position.

## Solution



1 mark for angle drawn in Quadrant IV
1 mark

Evaluate:

$$
\csc \left(\frac{11 \pi}{6}\right)+\sin ^{2}\left(-\frac{3 \pi}{4}\right)+\cos \left(\frac{23 \pi}{3}\right)
$$

## Solution

$$
\begin{array}{ll}
=(-2)+\left(-\frac{\sqrt{2}}{2}\right)^{2}+\frac{1}{2} & \\
=-2 \text { mark for } \csc \left(\frac{11 \pi}{6}\right)(1 / 2 \text { mark for quadrant, } 1 / 2 \text { mark for value }) \\
=-1 & \\
1 \text { mark for } \sin ^{2}\left(-\frac{1}{2}\right) \\
& 1 \text { mark for } \cos \left(\frac{23 \pi}{3}\right)(1 / 2 \text { mark for quadrant, } 1 / 2 \text { mark for quadrant, } 1 / 2 \text { mark for value }) \\
& \\
& 3 \text { marks }
\end{array}
$$

## June 2013

## Question 14

Angle $\theta$, measuring $\frac{5 \pi}{4}$, is drawn in standard position as shown below.
Determine the measures of all angles in the interval $[-4 \pi, 2 \pi]$ that are coterminal with $\theta$.

## Solution

$\theta=-\frac{3 \pi}{4} \quad 1 / 2 \mathrm{mark}$
$\theta=\frac{11 \pi}{4} \quad 1 / 2 \mathrm{mark}$
1 mark

## Question 1 (Calculator) <br> 2 marks

A central angle of a circle subtends an arc length of $5 \pi \mathrm{~cm}$. Given the circle has a radius of 9 cm , find the measure of the central angle in degrees.

## Solution

$$
\begin{aligned}
s & =\theta r & & \\
5 \pi & =\theta(9) & & 1 / 2 \text { mark for substitution into corre } \\
\theta & =\frac{5 \pi}{9} & & 1 / 2 \text { mark for solving for } \theta \\
\theta \text { (in degrees) } & =\frac{5 \pi}{9} \cdot \frac{180^{\circ}}{\pi} & & \\
& =100^{\circ} & & 1 \text { mark for conversion to degrees }
\end{aligned}
$$

Given the graph of $y=2 \cos \pi x+1$ below, determine another equation that will produce the same graph.


## Solution

Some sample equations are:

$$
\begin{aligned}
& y=2 \cos \pi(x-2)+1 \\
& y=-2 \cos \pi(x-1)+1 \\
& y=-2 \cos \pi(x+1)+1 \\
& y=2 \sin \pi\left(x+\frac{1}{2}\right)+1 \\
& y=2 \sin \pi\left(x-\frac{3}{2}\right)+1
\end{aligned}
$$

Other answers are possible.

## Question 27

2 marks
Explain how to find the exact value of $\sec \left(\frac{19 \pi}{6}\right)$.

## Solution

Find the exact value of $\cos \left(\frac{19 \pi}{6}\right)$.
1 mark for $\cos \left(\frac{19 \pi}{6}\right)$
Then take the reciprocal of the value of $\cos \left(\frac{19 \pi}{6}\right)$. 1 mark for reciprocal


## Question 32

The terminal arm of an angle $\theta$, in standard position, intersects the unit circle in Quadrant IV at a point $P\left(\frac{\sqrt{5}}{4}, y\right)$. Determine the value of $\sin \theta$.

## Solution

## Method 1

The point $\mathrm{P}(\theta)$ on the unit circle has coordinates $(\cos \theta, \sin \theta)$.

$$
\begin{aligned}
\cos ^{2} \theta+\sin ^{2} \theta & =1 & & 1 / 2 \text { mark for showing } y=\sin \theta \\
\left(\frac{\sqrt{5}}{4}\right)^{2}+\sin ^{2} \theta & =1 & & 1 / 2 \text { mark for substitution } \\
\sin ^{2} \theta & =1-\frac{5}{16} & & \\
\sqrt{\sin ^{2} \theta} & =\sqrt{\frac{11}{16}} & & \\
\sin \theta & = \pm \frac{\sqrt{11}}{4} & & 1 / 2 \text { mark for solving for } \sin \theta \\
\sin \theta & =-\frac{\sqrt{11}}{4} & & 1 / 2 \text { mark for a negative } \sin \theta \text { value in Quadrant IV }
\end{aligned}
$$

## Method 2

$$
\begin{array}{rlrl}
(\sqrt{5})^{2}+y^{2} & =4^{2} & & \\
5+y^{2} & =16 & & \\
y^{2} & =11 / 2 \text { mark for substitution } \\
y & = \pm \sqrt{11} & & \begin{array}{l}
1 / 2 \text { mark for solving for } y \\
\sin \theta
\end{array} \\
=-\frac{\sqrt{11}}{4} & & \begin{array}{l}
1 / 2 \text { mark for using the value of } y \text { to find the value of } \sin \theta \\
1 / 2 \text { mark for a negative } \sin \theta \text { value in Quadrant } I V
\end{array} \\
& & 2 \text { marks }
\end{array}
$$

Question 42
Sketch the graph of $y=10 \cos \left[\frac{\pi}{2}(x-2)\right]$ over the interval $[0,6]$.

## Solution

period $=\frac{2 \pi}{\frac{\pi}{2}}=4$


1 mark for amplitude
1 mark for period
1 mark for horizontal shift
3 marks

