Trigonometry: T1, T2, T3

$$\sin\theta = \frac{y\ (opp.)}{r\ (hyp.)} \quad \cos\theta = \frac{x\ (adj.)}{r\ (hyp.)} \quad \tan\theta = \frac{y\ (opp.)}{x\ (adj.)}$$

Use Pythagorus to find the 3 trigonometric ratios when given a point.

$$x^2 + y^2 = r^2$$

When solving for an angle:

- **Determine the reference angle (θ_r)**
- **Use the CAST rule to decide which quadrants contain the terminal arm**
- **Use the reference angle to calculate the angles:**
  - **QII** ($\theta = 180° - \theta_r$)
  - **QIII** ($\theta = 180° + \theta_r$)
  - **QIV** ($\theta = 360° - \theta_r$)

Sine Law: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

- **Ambiguous Case**
  - **Two solutions**
  - **One solution** (90°)
  - **No solution (Error)**

Cosine Law: $c^2 = a^2 + b^2 - 2ab\cos C$

Know your Unit Circle!!

1. $P(12, -3)$ is a point on the terminal arm of angle θ. Find the primary trigonometric ratios for angle θ.

$x^2 + y^2 = r^2$

$12^2 + (-3)^2 = r^2$

$144 + 9 = r^2$

$\pm\sqrt{153} = \sqrt{r^2}$

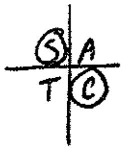
$\sqrt{153} = r$

$\sin\theta = \frac{y}{r} = \frac{-3}{\sqrt{153}}$

$\cos\theta = \frac{x}{r} = \frac{12}{\sqrt{153}}$

$\tan\theta = \frac{y}{x} = \frac{-3}{12} = \frac{-1}{4}$

2. Solve: $\tan\theta = \frac{-4}{3}$, $0 \leq \theta \leq 360°$



$\theta = \tan^{-1}(4 \div 3)$

$\theta r = 53.130°$

QII

$\theta = 180° - 53.130°$

$\theta = 126.870°$

QIV

$\theta = 360° - 53.130°$

$= 306.870°$

$\therefore \theta = 126.870°, 306.870°$

3. Solve: $\sin\theta = -\frac{\sqrt{3}}{2}$, $0 \leq \theta \leq 360°$

↳ on unit circle

$\theta = 240°, 300°$

4. Use your unit circle to find the exact value of the following:

a) $\cos 225°$ $-\frac{\sqrt{2}}{2}$

c) $\tan 330°$ $-\frac{1}{\sqrt{3}}$

b) $\sin 120°$ $\frac{\sqrt{3}}{2}$

d) $\sin 180°$ 0

5. Determine the angles in standard position for each quadrant that have a reference angle of 35°.

QI

$\theta = 35°$

QII

$\theta = 180° - 35°$

$= 145°$

QIII

$\theta = 180° + 35°$

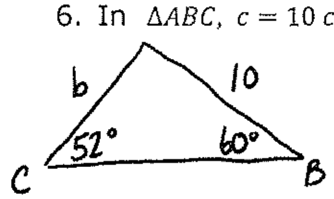
$= 215°$

QIV

$\theta = 360° - 35°$

$= 325°$

6. In ΔABC, $c = 10\ cm$, $< C = 52°$, and $< B = 60°$. Find side b.

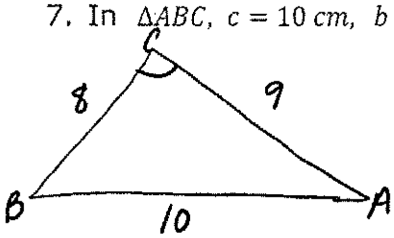


$\frac{b}{\sin 60°} = \frac{10}{\sin 52°}$

$b = \frac{10(\sin 60°)}{\sin 52°}$

$b = 10.990\ cm$

7. In $\Delta ABC,\ c = 10\,cm,\ \ b = 9\,cm,\ and\ \ a = 8\,cm.$ Find $< C$.



$$10^2 = 8^2 + 9^2 - 2(8)(9)\cos C$$

$$100 = 145 - 144\cos C$$

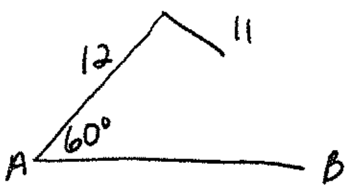
$$-145 \quad -145$$

$$\frac{-45}{-144} = \frac{-144\cos C}{-144}$$

$$0.3125 = \cos C$$

$$71.790° = \angle C$$

8. In $\Delta ABC, \angle A = 60°, a = 11, b = 12$. Solve for $\angle B$.



Ambiguous Case

$$\frac{11}{\sin 60°} = \frac{12}{\sin B}$$

$$\sin B = \frac{12(\sin 60°)}{11}$$

$$\sin B = 0.945$$

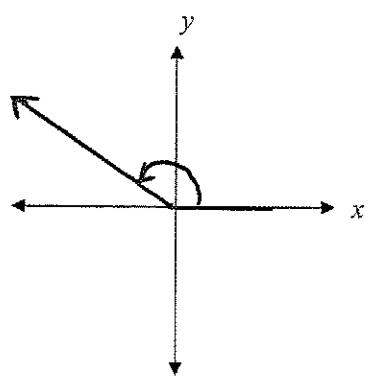
$$\angle B = 70.866°$$

$$\angle B = 180° - 70.866° = 109.134°$$

$\therefore \angle B = 70.866°$
$109.134°$

9. Draw the following angles in standard position.

a) 135°



b) $-305°$

$-305° + 360° = 55°$