Pre-Calculus 12 Enriched
Transformations Challenge

1. The function $f(x)$ satisfies the equation $f(x) = f(x-1) + f(x+1)$ for all values of x. Define $f(1) = 1$ and $f(3) = 3$; then, $f(2) = 1 + 3 = 4$. Determine the value of $f(1867)$.

2. For *x* and *k* real numbers, determine at what values of *k* the graphs of $f(x) = -2\sqrt{x+1}$ and $g(x) = \sqrt{x-2} + k$ will intersect.

3. The equation $y = x^2 + 2ax + a$ represents a parabola for all real values of *a*.
   a.) Prove that each of these parabolas passes through a common point and determine the coordinates of this point.
   b.) The vertices of the parabolas in part a) above lie on a curve. Prove that this curve is itself a parabola whose vertex is the common point found in part a.)

4. A **fixed point** of a function is an element of the domain that is mapped to itself by the function. That is, if $f(x)$ is a function and $f(p) = p$, then p is a fixed point of the function $f(x)$. Determine the fixed points, if any, of the following functions (assume *x* is an element of the real numbers):
   a.) $f(x) = 4x^2 - x - 6$
   b.) $f(x) = \frac{x}{1-x}$
   c.) $f(x) = |x| - 2$

5. **Floor and Ceiling Functions:**
   Define the *floor* for a real number, *x*, to be the largest integer $\leq x$. We write the floor of *x* as $\lfloor x \rfloor$.
   Define the *ceiling* for a real number, *x*, to be the smallest integer $\geq x$. We write the ceiling of *x* as $\lceil x \rceil$.
   a.) If $f(x) = \lfloor x \rfloor$, determine the values of $f(8.6), f(2), f(-2.6)$.
   b.) If $g(x) = \lceil x \rceil$, determine the values of $g(8.6), g(2), g(-2.6)$.

6. **Indicator Function** An *Indicator function*, $f(x)$, is a function that takes the value 1 if some condition on *x* is true, and the value 0 if the condition is not true. For integers *x*, define an indicator function:

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is divisible by } 2 \\ 0 & \text{if } x \text{ is not divisible by } 2 \end{cases}$$

and define another indicator function:

$$g(x) = \begin{cases} 1 & \text{if } x \text{ is divisible by } 3 \\ 0 & \text{if } x \text{ is not divisible by } 3 \end{cases}$$

a.) Determine the values of $f(2), f(9), f(4086), g(2), g(9), g(4086)$.
b.) Let $h(x) = f(x)g(x)$. Determine whether $h(x)$ is an indicator function, and, if so, describe in words the condition it indicates.

7. **Modulo Function** The function $m_k(x)$has as its domain the non-negative integers. It is defined such that $m_k(x)$is the smallest non-negative remainder when x is divided by the positive integer k. For example, $m_4(14) = 2$; $m_{13}(42) = 3$.
   a.) Determine the values of $m_9(101), m_6(720), m_7(55)$.
   b.) State the range of the function $m_k(x)$.