Squeeze Theorem

Assume that functions f, g and h satisfy

$$g(x) \le f(x) \le h(x)$$

and

$$\lim_{x\to A} g(x) = L = \lim_{x\to A} h(x)$$

then

$$\lim_{x\to A} f(x) = L$$

ex. 1. Evaluate

$$\lim_{x\to\infty} \frac{\sin x}{x}$$

Note $\quad -1 \le \sin x \le 1$

since we are evaluating the limit as x approaches infinity it is reasonable to divide by x

$$-\frac{1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$$

since $\lim_{x\to\infty}\left(-\frac{1}{x}\right) = 0 = \lim_{x\to\infty}\left(\frac{1}{x}\right)$

$\therefore$ it follows from the squeeze theorem

$$\lim_{x\to\infty} \frac{\sin x}{x} = 0$$

ex 2 Evaluate $\lim_{x\to\infty}\left(\frac{2-\cos x}{x+3}\right)$

$$-1 \le \cos x \le 1$$

multiply by -1, flip the inequalities

$$1 \ge -\cos x \ge -1$$

or $\quad \overset{+2}{-1} \overset{+2}{\le} -\cos x \le \overset{+2}{1}$

$$1 \le 2-\cos x \le 3$$

since we are evaluating the limit as x approaches infinity it is reasonable to divide by x+3

$$\frac{1}{x+3} \le \frac{2-\cos x}{x+3} \le \frac{3}{x+3}$$

Since

$$\lim_{x\to\infty} \frac{1}{x+3} = 0 = \lim_{x\to\infty} \frac{3}{x+3}$$

$$\therefore \lim_{x\to\infty} \frac{2-\cos x}{x+3} = 0$$

Evaluate $\lim_{x\to-\infty} \frac{5x^2-\sin(3x)}{x^2+10}$

$$-1 \le \sin(3x) \le 1$$

or

$$-1 \le -\sin(3x) \le 1$$

add $5x^2$

$$5x^2-1 \le 5x^2-\sin(3x) \le 5x^2+1$$

since we are evaluating the limit as x approaches $-\infty$ it is reasonable to divide by x^2+10

$$\frac{5x^2-1}{x^2+10} \le \frac{5x^2-\sin(3x)}{x^2+10} \le \frac{5x^2+1}{x^2+10}$$

since

$$\lim_{x\to-\infty} \frac{5x^2-1}{x^2+10} = 5 = \lim_{x\to-\infty} \frac{5x^2+1}{x^2+10}$$

$$\therefore \lim_{x\to-\infty} \frac{5x^2-\sin(3x)}{x^2+10}$$

Evaluate

1) $\lim_{x\to\infty} \frac{\cos^2(2x)}{3-2x}$

2) $\lim_{x\to 0^-} x^3\cos\left(\frac{2}{\pi}\right)$

3) $\lim_{x\to\infty} \frac{x^2(2+\sin^2 x)}{x+100}$