The Quotient Rule

If $h(x) = \dfrac{f(x)}{g(x)}$

then $h'(x) = \dfrac{g(x)\,f'(x) - f(x)\,g'(x)}{(g(x))^2}$

or

$$\frac{\text{lo d'hi} - \text{hi d'lo}}{\text{the bottom all squared}}$$

ex.1 Find $f'(x)$

a) $f(x) = \dfrac{x^2+1}{x^3+2}$

$$f'(x) = \frac{(x^3+2)(2x) - (x^2+1)(3x^2)}{(x^3+2)^2}$$

$$f'(x) = \frac{2x^4 + 4x - 3x^4 - 3x^2}{(x^3+2)^2}$$

$$= \frac{-x^4 - 3x^2 + 4x}{(x^3+2)^2}$$

b) $f(x) = \dfrac{x^2+x-2}{x^3+6}$

$$f'(x) = \frac{(x^3+6)(2x+1) - (x^2+x-2)(3x^2)}{(x^3+6)^2}$$

$$= \frac{2x^4 + x^3 + 12x + 6 - 3x^4 - 3x^3 + 6x^2}{(x^3+6)^2}$$

$$= \frac{-x^4 - 2x^3 + 6x^2 + 12x + 6}{(x^3+6)^2}$$

ex.2 Differentiate

$f(x) = \dfrac{3x^2 + 2\sqrt{x}}{x}$

$$f(x) = \frac{3x^2 + 2x^{\frac{1}{2}}}{x}$$

$$f(x) = \frac{3x^2}{x} + \frac{2x^{\frac{1}{2}}}{x}$$

$$f(x) = 3x + 2x^{-\frac{1}{2}}$$

$$f'(x) = 3 - x^{-\frac{3}{2}}$$

Sometimes simplifying eliminates the need for the quotient rule.

or

$$f'(x) = \frac{x(6x + x^{-\frac{1}{2}}) - (3x^2 + 2\sqrt{x})(1)}{x^2}$$

$$= \frac{6x^2 + x^{\frac{1}{2}} - 3x^2 - 2x^{\frac{1}{2}}}{x^2}$$

$$= \frac{3x^2 - x^{\frac{1}{2}}}{x^2} \leftarrow \text{can stop here}$$

$$= \frac{3x^2}{x^2} - \frac{x^{\frac{1}{2}}}{x^2}$$

$$= 3 - x^{-\frac{3}{2}}$$

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Try 19

Note:

$y = 3x^2$

$\frac{dy}{dx} = 6x$

or

$y' = 6x$

$f(x) = 3x^2$

$f'(x) = 6x$

$3x^2$

$6x$

$\frac{d}{dx}(3x^2) = 6x$

ex.2 Differentiate

$$f(x) = \frac{3x^2 + 2\sqrt{x}}{x}$$

$$f'(x) = \frac{x(6x + x^{-\frac{1}{2}}) - (3x^2 + 2x^{\frac{1}{2}})}{x^2}$$

$$= \frac{6x^2 + x^{\frac{1}{2}} - 3x^2 - 2x^{\frac{1}{2}}}{x^2}$$

$$= \frac{3x^2 - x^{\frac{1}{2}}}{x^2}$$

$$= \frac{3x^2}{x^2} - \frac{x^{\frac{1}{2}}}{x^2}$$

$$= 3 - x^{-\frac{3}{2}}$$