The Product Rule

definition of the Product Rule

If $f(x) = g(x) \cdot h(x)$, then ...

$$f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$$

or easier on "cheat sheet" $\frac{d}{dx}(uv) = u'v + v'u$

ex. 1 Find the derivative.

a) $f(x) = x^2(x+1)$ (u = x^2, v = $x+1$)

$= x^3 + x^2$

$f'(x) = 3x^2 + 2x$

OR

Using the product rule $f'(x) = u'v + v'u$

$f'(x) = 2x(x+1) + (1)(x^2)$

$= 2x^2 + 2x + x^2$

$= 3x^2 + 2x$

b) $f(x) = (2x^3+3)(x^4-2x)$

$f'(x) = 6x^2(x^4-2x) + (4x^3-2)(2x^3+3)$

$= 6x^6 - \cancel{12x^3} + 8x^6 + \cancel{12x^3} - 4x^3 - 6$

$f'(x) = 14x^6 - 4x^3 - 6$

c) $f(t) = \sqrt{t}(1-t)$

$= t^{\frac{1}{2}}(1-t)$

$f'(t) = \frac{1}{2}t^{-\frac{1}{2}}(1-t) + (-1)(t^{\frac{1}{2}})$

$= \frac{1}{2}t^{-\frac{1}{2}} - \frac{1}{2}t^{\frac{1}{2}} - t^{\frac{1}{2}}$

$= \frac{1}{2}t^{-\frac{1}{2}} - \frac{3}{2}t^{\frac{1}{2}}$

$u = t^{\frac{1}{2}}$
$u' = \frac{1}{2}t^{-\frac{1}{2}}$
$v = 1-t$
$v' = -1$
then sub $u'v + v'u$

Note: Both b) and c) could have been done using the distributive property and the power rule!

ex. 2 Find an eqn. for the line tangent to the curve $y = x^2$ at the point $(2,4)$

$y = x^2$

$\frac{dy}{dx} = 2x$

at $x = 2$ $m = 2(2)$
$= 4$

$y - y_1 = m(x - x_1)$

$y - 4 = 4(x-2)$

or

$y - 4 = 4x - 8$

$0 = 4x - y - 4$

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