The Derivative of a Function

Recall:

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

when it exists this limit is called the derivative of f at a.

Def'n
The derivative of the function f w.r.t. the variable x is the fcn f whose value is

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

provided the limit exists

Notations

y' $f'(x)$ $\frac{dy}{dx}$ $D_x y$ $\frac{df(x)}{dx}$

ex. Find the derivative of the given fcn. using the definition of the derivative.

a) $f(x) = 2x^3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$2(x+\Delta x)(x+\Delta x)^2$

$$= \lim_{\Delta x \to 0} \frac{2(x+\Delta x)^3 - 2x^3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(2x+2\Delta x)(x^2 + 2x\Delta x + \Delta x^2) - 2x^3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{2x^3} + 6x^2\Delta x + 6x\Delta x^2 + 2\Delta x^3 - \cancel{2x^3}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(6x^2 + 6x\Delta x + 2\Delta x^2)}{\cancel{\Delta x}}$$

$$f'(x) = 6x^2 + 6x(0) + 2(0)^2$$

$$f'(x) = 6x^2$$

Shortcut:

$f(x) = 2x^3$

$f'(x) = 2 \cdot 3x^{3-1}$

$= 6x^2$

b) $f(x) = 3x^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{3(x+h)^2 - 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) - 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{3x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(6x + 3h)}{\cancel{h}}$$

$$= 6x + 3(0)$$

$$= 6x$$

Def'n of Derivative sheet
odds

~~Pg 105 Exercises # 3, 6, 8, 10~~

The Power Rule

$$\frac{d}{dx} x^n = n \cdot x^{n-1}$$

$$y = x^n$$

$$\frac{dy}{dx} = n \cdot x^{n-1}$$

The Power Rule

ex. 1 State the derivative of:

a) $10x$

10

$10x^{1-1}$

$1 \cdot 10\cancel{x}^{\cancel{0}}$

$\frac{d}{dx}(10x) = 10$

b) $6x^3 - 4x^2 + 5x$

$18x^2 - 8x + 5$

c) 27

0

$27x^0$

$\cancel{0 \cdot 27x^{0-1}}$

0

d) $y = \frac{1}{x^3}$

$y' = x^{-3}$

$y' = -3x^{-4}$ or $\frac{-3}{x^4}$

e) $\frac{5x^3 + x^2}{x}$

$\frac{\cancel{x}(5x^2 + x)}{\cancel{x}}$

$5x^2 + x$

$10x + 1$

$\frac{d}{dx}\left(\frac{5x^3 + x^2}{x}\right) = 10x + 1$

or if given Notation

$f(x) = -3x^4 - x + 5$

$f'(x) = -12x^3 - 1$

Recall

$\sqrt{x} = x^{\frac{1}{2}}$