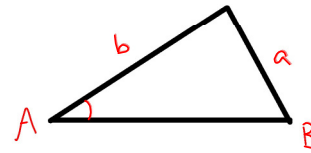


## Lesson 5 The Ambiguous Case

The ambiguous case occurs when you are given two sides and an angle of a triangle (SSA) and the angle is opposite the shorter side.

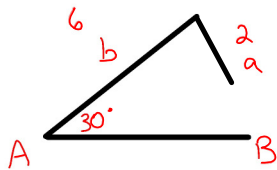
**There are two cases:**

1. If  $a > b$ , then only one solution exists.
2. If  $a < b$ , then you have three possibilities
  - a) No triangle is formed (No Solution)
  - b) A Right triangle is formed (One Solution)
  - c) Two Triangles are formed (Two Solutions)



### **Example of Cases with No Solution**

1. Determine the measure of  $\angle B$  given  $\triangle ABC$ , where  $a = 2$ ,  $b = 6$ , and  $\angle A = 30^\circ$ .



$$\frac{2}{\sin 30^\circ} = \frac{6}{\sin B}$$

$$\sin B = \frac{6 \sin 30^\circ}{2}$$

$$\sin B = 1.5$$

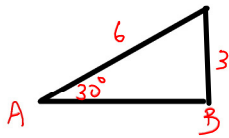
$\emptyset$

$\therefore$  no triangle exists

$$-1 \leq \sin B \leq 1$$

**Examples of Cases with One Solution**

1. Determine the measure of  $\angle B$  given  $\triangle ABC$ , where  $a = 3$ ,  $b = 6$ , and  $\angle A = 30^\circ$ .



$$\frac{3}{\sin 30^\circ} = \frac{6}{\sin B}$$

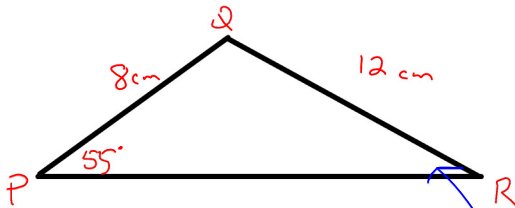
$$\sin B = \frac{6 \sin 30^\circ}{3}$$

$$\sin B = 1$$

$$B = 90^\circ$$

$\therefore$  the triangle is a right triangle

2. In  $\triangle PQR$ ,  $PQ = 8$  cm,  $\angle P = 55^\circ$ , and  $QR = 12$  cm. Solve the triangle.



$12 > 8 \therefore$  one triangle exists

$$\textcircled{1} \frac{12}{\sin 55^\circ} = \frac{8}{\sin R}$$

$$\sin R = \frac{8 \sin 55^\circ}{12}$$

$$R = \sin^{-1}(\text{ans})$$

$$= 33.09995\dots$$

$$= 33.100^\circ$$

$$\textcircled{2} Q = 180^\circ - (55^\circ + 33.0995\dots)$$

$$= 91.9000\dots$$

$$= 91.900^\circ$$

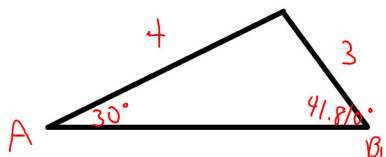
$$\textcircled{3} \frac{12}{\sin 55^\circ} = \frac{2}{\sin 91.9000\dots}$$

$$2 = \frac{12 \sin 91.9000\dots}{\sin 55^\circ}$$

$$= 14.641 \text{ cm}$$

**Examples of Cases with Two Solutions**

1. Determine the measure of  $\angle B$  given  $\triangle ABC$ , where  $a = 3$ ,  $b = 4$ , and  $\angle A = 30^\circ$ .



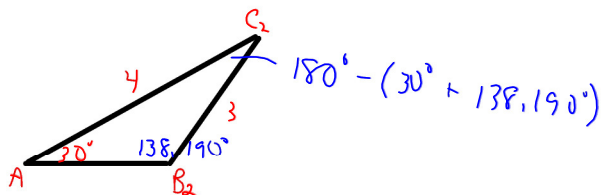
acute  $\angle$

$$\frac{3}{\sin 30^\circ} = \frac{4}{\sin B_1}$$

$$\sin B_1 = \frac{4 \sin 30^\circ}{3}$$

$$* B_1 = \sin^{-1}(\text{ans})$$

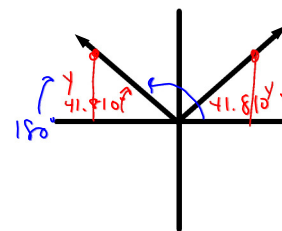
$$= 41.810^\circ$$



$$180^\circ - (30^\circ + 138.19^\circ)$$

$$B_2 = 180^\circ - 41.810^\circ$$

$$= 138.190^\circ$$



# 1, 3, 5, 7, 9

# 4 challenge

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worksheet # 2, 4, 6, 8, 10

and/or

pg 145 # 6 k, l, n, o, q, r  
8, 9