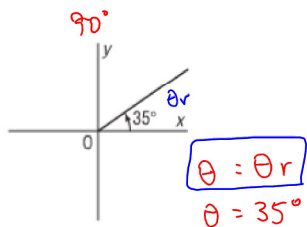


Lesson 2 Reference Angles

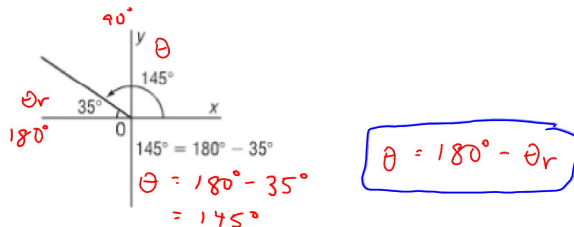
A **reference angle** is the positive acute angle formed between the terminal arm of the angle and the x-axis.

$$\hookrightarrow 0^\circ < \theta_r < 90^\circ$$

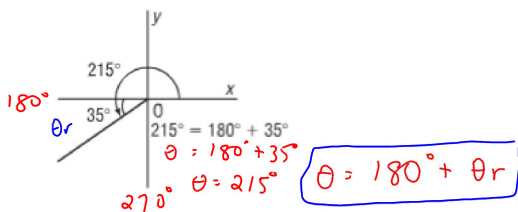
Quadrant I



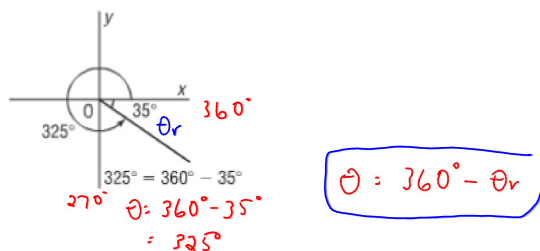
Quadrant II



Quadrant III



Quadrant IV



Examples

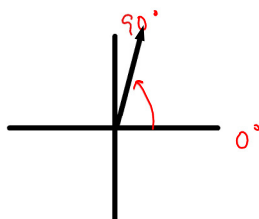
$\therefore 35^\circ, 145^\circ, 215^\circ, \text{ and } 325^\circ$ are related angles

1. Sketch the following angles in standard position and determine the reference angle.

a) 87°

$$\theta = \theta_r$$

$$\theta_r = 87^\circ$$

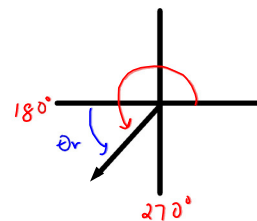


b) 232°

$$\theta = 180^\circ + \theta_r$$

$$232^\circ = 180^\circ + \theta_r$$

$$52^\circ = \theta_r$$



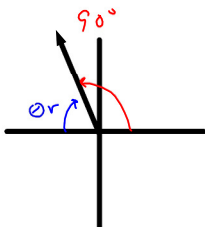
c.) 101°

$$\theta = 180^\circ - \theta_r$$

$$101^\circ = 180^\circ - \theta_r$$

$$\theta_r = 180^\circ - 101^\circ$$

$$\theta_r = 79^\circ$$

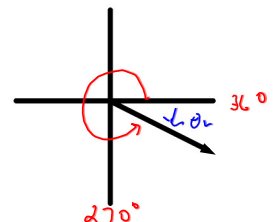


d.) 340°

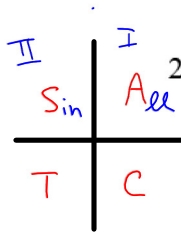
$$\theta = 360^\circ - \theta_r$$

$$340^\circ = 360^\circ - \theta_r$$

$$\theta_r = 20^\circ$$



L2 Reference Angles.notebook



2. Determine the values of θ such that $0^\circ \leq \theta \leq 360^\circ$ for the given trigonometric functions. Express answers to the nearest tenth of a degree.

a) $\sin \theta = 0.78615$

$$\theta_r = \sin^{-1}(0.78615)$$

$$\theta_r = 51.8^\circ$$

$\sin \theta > 0$
in Quads I, II

QI $\theta = \theta_r$
 $\theta = 51.8^\circ$

QII $\theta = 180^\circ - \theta_r$
 $= 180^\circ - 51.8^\circ$
 $= 128.2^\circ$

b) $\cos \theta = 0.43214$

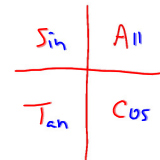
$$\theta_r = \cos^{-1}(0.43214)$$

$$= 64.4^\circ$$

$\cos \theta > 0$
in QI, IV

QI $\theta = 64.4^\circ$

QIV $\theta = 360^\circ - \theta_r$
 $= 360^\circ - 64.4^\circ$
 $= 295.6^\circ$



c) $\tan \theta = -2.81763$

$$\theta_r = \tan^{-1}(+2.81763)$$

$$\theta_r = 70.5^\circ \leftarrow \text{not a sol'n}$$

Always use a +ve value to get the reference angle

$\tan \theta < 0$
in QII, IV

QII $\theta = 180^\circ - \theta_r$
 $= 180^\circ - 70.5^\circ$
 $\theta = 109.5^\circ$

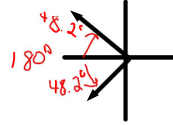
QIV $\theta = 360^\circ - 70.5^\circ$
 $\theta = 289.5^\circ$

sol'n's

L2 Reference Angles.notebook

3. Determine the solution set for each of the following trigonometric equations over the interval $0^\circ \leq \theta \leq 360^\circ$.

a) $\cos\theta = -\frac{2}{3}$
 $\theta_r = \cos^{-1}\left(-\frac{2}{3}\right)$
 $\theta_r = 48.2^\circ$



$\cos\theta < 0$
in QII, III

QII $\theta = 180^\circ - 48.2^\circ$
 $= 131.8^\circ$

QIII

$\theta = 180^\circ + 48.2^\circ$
 $= 228.2^\circ$

b) $\tan\theta = -\frac{4}{3}$
 $\theta_r = \tan^{-1}\left(\frac{4}{3}\right)$
 $\theta_r = 53.1^\circ$

$\tan\theta < 0$
in QII, IV

QII $\theta = 180^\circ - 53.1^\circ$
 $= 126.9^\circ$

QIV

$\theta = 360^\circ - 53.1^\circ$
 $= 306.9^\circ$

c) $-3\sin\theta = 2$
 $\sin\theta = -\frac{2}{3}$
 $\theta_r = \sin^{-1}\left(\frac{2}{3}\right)$
 $\theta_r = 41.8^\circ$

$\sin\theta < 0$
in QIII, IV

QIII $\theta = 180^\circ + 41.8^\circ$
 $= 221.8^\circ$

QIV

$\theta = 360^\circ - 41.8^\circ$
 $= 318.2^\circ$

pg. 126 #1a, b, g, h, i,
 2a, c, e
 4g, h, i
 5a, c, e, g