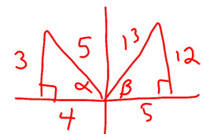
Lesson 4 Sum and Difference Identities…again

$\cos\beta > 0$ and $\therefore$ QI

Ex. 1) Given $\sin \alpha = \frac{3}{5}$ with α in QII and $\cos\beta = \frac{5}{13}$ with $\tan\beta > 0$, determine the exact value of

a) $\sin(\alpha + \beta)$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$= \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{-4}{5}\right)\left(\frac{12}{13}\right)$$
$$= \frac{15}{65} - \frac{48}{65}$$
$$= \frac{-33}{65}$$



$\cos\alpha = \frac{-4}{5}$ $\sin\beta = \frac{12}{13}$

3, 4, 5
5, 12, 13 triplets
or
use pythagorean thm

b) $\cos(\alpha + \beta)$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$= \left(\frac{-4}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{12}{13}\right)$$
$$= \frac{-20}{65} - \frac{36}{65}$$
$$= \frac{-56}{65}$$

c) $\tan(\alpha + \beta)$

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$$
$$= \frac{\frac{-33}{\cancel{65}}}{\frac{-56}{\cancel{65}}}$$
$$= \frac{33}{56}$$

d) The coordinates of $P(\alpha + \beta)$

$$(\cos(\alpha+\beta), \sin(\alpha+\beta))$$
$$\left(-\frac{56}{65}, \frac{-33}{65}\right)$$

in quadrant III

Pre-Calculus 12 Enriched Trigonometric Equations & Identities

Ex. 2) Using $\cos\left(\frac{\pi}{2}+\frac{\pi}{2}\right)$, verify that $\cos\pi = -1$.

$\cos\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = \cos\frac{\pi}{2}\cos\frac{\pi}{2} - \sin\frac{\pi}{2}\sin\frac{\pi}{2}$

$= 0 \cdot 0 - 1 \cdot 1$

$\cos(\pi) = -1$

Ex. 3) Prove $\sin(\pi - x) = \sin x$

diff identity

Left-Hand Side	Right-Hand Side
$\sin\pi\cos x - \cos\pi\sin x$	$\sin x$
$\cancel{0\cos x} - (-1)\sin x$	
$\sin x$	

LHS = RHS ✓

Ex. 4) Prove $\sin(A+B) + \sin(A-B) = 2\sin A\cos B$

Left-Hand Side	Right-Hand Side
$\sin A\cos B + \cancel{\cos A\sin B} + \sin A\cos B - \cancel{\cos A\sin B}$	$2\sin A\cos B$
$2\sin A\cos B$	

LHS = RHS ✓

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\# 2d, f, h
3a, c, d, e, f
5a, e, d