Lesson 3 Sum and Difference Identities

Sum Identities

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

Difference Identities

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha-\beta) = \dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

We use these to determine exact values of angles not on the special circle.

Ex. 1) Determine the exact value of $\cos\frac{7\pi}{12}$.

$\frac{7\pi}{12} = \frac{3\pi}{12} + \frac{4\pi}{12}$

$= \frac{\pi}{4} + \frac{\pi}{3}$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$= \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right)$

$\cos\frac{7\pi}{12} = \frac{\sqrt{2}-\sqrt{6}}{4}$ ← exact value

Ex. 2) Determine the exact value of $\tan\frac{5\pi}{12}$.

sum $\frac{5\pi}{12} = \frac{2\pi}{12} + \frac{3\pi}{12}$

$= \frac{\pi}{6} + \frac{\pi}{4}$

or

diff $\frac{5\pi}{12} = \frac{8\pi}{12} - \frac{3\pi}{12}$

$= \frac{2\pi}{3} - \frac{\pi}{4}$

$\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$\tan\left(\frac{\pi}{6}+\frac{\pi}{4}\right) = \dfrac{\tan\frac{\pi}{6}+\tan\frac{\pi}{4}}{1-\tan\frac{\pi}{6}\tan\frac{\pi}{4}}$

$= \dfrac{\frac{\sqrt{3}}{3}+1\left(\frac{3}{3}\right)}{\left(\frac{3}{3}\right)1-\frac{\sqrt{3}}{3}(1)}$

$= \dfrac{\frac{\sqrt{3}+3}{3}}{\frac{3-\sqrt{3}}{3}}$

$= \frac{\sqrt{3}+3}{3-\sqrt{3}}$

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Ex. 3) Determine the exact value of $\sin\frac{3\pi}{2}\cos\frac{5\pi}{4} - \cos\frac{3\pi}{2}\sin\frac{5\pi}{4}$.

(α β α β)

← right hand side of $\sin(\alpha-\beta)$

$\sin(\alpha-\beta)$ ← left hand side

$\sin\left(\frac{3\pi}{2} - \frac{5\pi}{4}\right)$

$\sin\left(\frac{6\pi}{4} - \frac{5\pi}{4}\right)$

$\sin\frac{\pi}{4}$

$\frac{\sqrt{2}}{2}$

Ex. 4) Determine the exact value of $\cos\frac{\pi}{12}\cos\frac{\pi}{3} + \sin\frac{\pi}{12}\sin\frac{\pi}{3}$.

(α β α β)

$\cos(\alpha-\beta) = \cos\frac{\pi}{12}\cos\frac{\pi}{3} + \sin\frac{\pi}{12}\sin\frac{\pi}{3}$

$\cos\left(\frac{\pi}{12} - \frac{\pi}{3}\right)$

$\cos\left(\frac{\pi}{12} - \frac{4\pi}{12}\right)$

$\cos\left(-\frac{3\pi}{12}\right)$

$\cos\left(-\frac{\pi}{4}\right)$

$\frac{\sqrt{2}}{2}$

Ex. 5) Express $\cos\left(\frac{\pi}{2}+x\right)$ as a function of x only.

$\cos\left(\frac{\pi}{2}+x\right) = \cos\frac{\pi}{2}\cos x - \sin\frac{\pi}{2}\sin x$

$= 0\cos x - 1\sin x$

$= -\sin x$

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1 a, e
2 a, c, e