Lesson 5 Infinite Geometric Series

An ***infinite geometric series*** has an infinite number of terms. As the terms get closer and closer to some real number, the series is said to be convergent and will converge to a real number.

This sum is called the sum to infinity and is denoted by S_∞.

Deriving the Sum Formula for an Infinite Geometric Series

$S_\infty = \frac{t_1}{1-r}$

where
t_1 is the first term
r is the common ratio
$|r| < 1$ or $-1 < r < 1$

Examples

1. Determine whether each infinite geometric series converges or diverges.

   a) $\frac{1}{3} + \frac{1}{12} + \frac{1}{48} + \frac{1}{192} + \ldots$

   b) $-4 - 8 - 16 - 32 - \cdots$

c) $\frac{1}{10} - \frac{1}{100} + \frac{1}{1000} - \frac{1}{10\,000} + \ldots$

2. Determine the sum of each geometric series, if it exists.

a) $32 + 8 + 2 + \frac{1}{2} + \ldots$

b) $100 - 10 + 1 - \frac{1}{10} + \ldots$

c) $1 + 8 + 64 + \ldots$

3. The sum of an infinite geometric series is 16. If the common ratio is $\frac{1}{2}$, find the value of t_1.

4. The infinite series given by $1 + 3x + 9x^2 + 27x^3 + \cdots$ has a sum of 4. Determine the value of x.

5. A ball is dropped from 12 ft and rebounds $\frac{2}{3}$ of the distance from which it fell. Determine the total distance the ball travelled.