Lesson 5 Infinite Geometric Series

An ***infinite geometric series*** has an infinite number of terms. As the terms get closer and closer to some real number, the series is said to be convergent and will converge to a real number.

This sum is called the sum to infinity and is denoted by S_∞.

Deriving the Sum Formula for an Infinite Geometric Series

pg. 368

$$S_\infty = \frac{t_1}{1-r}$$

where
t_1 is the first term
r is the common ratio
$|r| < 1$ or $-1 < r < 1$

Examples

1. Determine whether each infinite geometric series converges or diverges.

a) $\frac{1}{3} + \frac{1}{12} + \frac{1}{48} + \frac{1}{192} + \ldots$

$r = \frac{1}{4}$ series converges $-1 < r < 1$

b) $-4 - 8 - 16 - 32 - \cdots$

$r = 2$ series diverges $|r| > 1$
$r < -1$ or $r > 1$

$t_1 = 10 \qquad r = \frac{1}{2}$

15 17.5 18.75 19.375 19.6875 19.84375 19.921875

$10 + 5 + 2.5 + 1.25 + 0.625 + 0.3125 + 0.15625 + 0.078125 +$

$0.0390625 + 0.01953125$

19.9609375 19.98046875

$$S_\infty = \frac{t_1}{1-r}$$

$$= \frac{10}{1-\frac{1}{2}}$$

$$= 20$$

c) $\frac{1}{10} - \frac{1}{100} + \frac{1}{1000} - \frac{1}{10\,000} + \ldots$

$-1 < r < 1$

$r = \frac{-1}{10}$

$\therefore$ series converges

2. Determine the sum of each geometric series, if it exists.

a) $32 + 8 + 2 + \frac{1}{2} + \ldots$

$S_\infty = \frac{t_1}{1-r}$

$r = \frac{1}{4}$

$= \frac{32}{1 - \frac{1}{4}}$

$-1 < \frac{1}{4} < 1$

$= 32 \cdot \frac{4}{3}$

$\therefore$ sum exists

$= \frac{128}{3}$

b) $100 - 10 + 1 - \frac{1}{10} + \ldots$

$S_\infty = \frac{100}{1-(-\frac{1}{10})}$

$r = -\frac{1}{10}$

$= \frac{100}{\frac{11}{10}}$

$-1 < -\frac{1}{10} < 1$

$\therefore$ sum exists

$= 100 \cdot \frac{10}{11}$

$= \frac{1000}{11}$

c) $1 + 8 + 64 + \ldots$

$r = 8$

$|r| > 1$

$\therefore$ infinite sum does not exist because series is divergent

3. The sum of an infinite geometric series is 16. If the common ratio is $\frac{1}{2}$, find the value of t_1.

$S_\infty = \frac{t_1}{1-r}$

$16 = \frac{t_1}{1-\frac{1}{2}}$

$8 = t_1$

4. The infinite series given by $1 + 3x + 9x^2 + 27x^3 + \cdots$ has a sum of 4. Determine the value of x.

$S_\infty = \frac{t_1}{1-r}$

$4 = \frac{1}{1-3x}$

$4 - 12x = 1$

$3 = 12x$

$\frac{1}{4} = x$

5. A ball is dropped from 12 ft and rebounds $\frac{2}{3}$ of the distance from which it fell. Determine the total distance the ball travelled.

vertical distance

12 ft

$12\left(\frac{2}{3}\right)$

$t_1 = 2(12)$

$= 24$

$r = \frac{2}{3}$

$S_\infty = \frac{t_1}{1-r}$

$= \frac{24}{1-\frac{2}{3}}$

$= 72$

since 12m drop didn't have a 12m bounce

we $72 - 12$

60 ft

12

$12\left(\frac{2}{3}\right)$

8 ft

sum of rebounds $= \frac{8}{1-\frac{2}{3}}$

$= 24$

each rebound has an identical drop

$2(24)$

and add intial drop

$48 + 12$

60 ft

Vert dist = 2 (sum of rebounds) + initial drop

pg. 371

1a, b, d

2 a, e, f

3 a, d

9

12

19, 20

Review

pg. 376