Lesson 4 Geometric Series

A geometric series is the sum of the terms of a geometric sequence.

Geometric sequence: $1, 3, 9, 27, \ldots$

Geometric Series: $1 + 3 + 9 + 27 + \cdots$

Deriving the Formula for the Partial Sum of n terms of a Geometric Series

If $S_n = t_1 + t_1r + t_1r^2 + \ldots + t_1r^{n-1}$ is a geometric series with common ratio, r, the sum can be found by multiplying S_n by $(-r)$ and adding the new equation to S_n

$$S_n = t_1 + \cancel{t_1r} + \cancel{t_1r^2} + \ldots + \cancel{t_1r^{n-1}}$$

$(-r)S_n$ $$-rS_n = -\cancel{t_1r} - \cancel{t_1r^2} - \ldots - t_1r^n$$

$$S_n - rS_n = t_1 - t_1r^n$$

$r^{n-1}r^1$

r^{n-1+1}

GCF $$S_n(1-r) = t_1 - t_1r^n$$

$$\therefore\ S_n = \frac{t_1 - t_1r^n}{1-r}$$

$$\boxed{S_n = \frac{t_1(1-r^n)}{1-r}}$$

or

$$S_n = \frac{t_1 - t_1r^n}{1-r}$$

$$= \frac{t_1 - t_1r^{n-1}r}{1-r}$$

$t_n = t_1r^{n-1}$

$$\boxed{S_n = \frac{t_1 - rt_n}{1-r}}$$

$$S_n = \frac{t_1(1-r^n)}{1-r}, r \neq 1$$

where: S_n is the sum of the first n terms
t_1 is the first term of the series
r is the common ratio
n is the number of terms

Examples

1. Determine the sum of the first 12 terms of the given geometric series.
   $3 + 12 + 48 + 192 + \cdots$

$t_1 = 3$
$n = 12$
$r = 4$

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

$$S_{12} = \frac{\cancel{3}(1-4^{12}))}{(\cancel{1-4})}$$

$$= -(1-4^{12})$$

$$= 16\ 777\ 215$$

2. The sum of the first 14 terms of a geometric series is 16 383. The common ratio is -2. Determine the value of the 1st term. solving for t_1

$S_{14} = 16\ 383$
$r = -2$
$n = 14$
$t_1 = ?$

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

$$16\ 383 = \frac{t_1(1-(-2)^{14})}{1-(-2)}$$

$$\cancel{16\ 383} = \frac{t_1(-\cancel{16\ 383})}{3}$$

$$3 = -t_1$$

$$-3 = t_1$$

or

$$16\ 383 = -5461\, t_1$$

$$-3 = t_1$$

3. Calculate the sum of the given geometric series.
   $-3 - 15 - 75 - \ldots - 46\ 875$

t_1 $r = 5$ t_n

$$S_n = \frac{t_1 - rt_n}{1-r}$$

$$= \frac{(-3 - 5(-46\ 875)))}{(1-5)}$$

$$= -58\ 593$$

4. A person takes tablets to cure a chest infection. Each tablet contains 500 mg of an antibiotic. About 15% of the mass of the antibiotic remains in the body when the next tablet is taken. Determine the mass of antibiotic in the body after each number of tablets.

a) 3 tablets

$S_1 = 500$

$$S_2 = \underset{\substack{\uparrow \\ \text{new tablet}}}{500} + \underbrace{500(0.15)}_{\text{15\% of 1st tablet remains}}$$

$$S_3 = \underset{\substack{\uparrow \\ \text{3rd tablet}}}{500} + \underbrace{500(0.15)}_{\text{15\% of 2nd tablet}} + \underbrace{500(0.15)^2}_{\text{1st tablet}}$$

$= 586.25\text{mg}$

b) 10 tablets

$t_1 = 500$

$r = 0.15$

$n = 10$

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

$$S_{10} = \frac{500(1-0.15^{10})}{1-0.15}$$

$= 588.24\text{mg}$

5. Evaluate.

$$\sum_{k=1}^{10} 3(-2)^{k-1}$$

variable in exponent

$k=1$
$t_1 = 3(-2)^{1-1}$
$= 3$

$k=10$
$n=10$

$$S_{10} = \frac{3(1-(-2)^{10})}{1-(-2)}$$

$$= -1023$$

6. Express the given geometric series in sigma notation with the index $k = 1$.
   $6 + 18 + 54 + 162 + 486$

$$\sum_{k=1}^{5} 6(3^{k-1})$$

models
$t_n = t_1 r^{n-1}$
$6(3)^{k-1}$

$$\underbrace{6(3^{1-1}) + 6(3^{2-1}) + 6(3^{3-1}) + 6(3^{4-1}) + 6(3^{5-1})}$$

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\# 1 b, c, f, h
2 a
3 a, c
4 a
7
19
Try 20