Lesson 3 Geometric Sequences

A *geometric sequence* is formed by multiplying each term after the 1ˢᵗ term by a *common ratio*, r, to determine the next term.

The common ratio, r, is any non-zero real number that can be determined by dividing any term by the preceding term.

3, 6, 12, 24 … has a common ratio of ____

3, 6, 12, 24 … is called an *infinite geometric sequence*

3, 6, 12, 24   is called a *finite geometric sequence*

Examples

1. Identify which of the following are geometric sequences.

   a) 3, 6, 9, 12, …

   b) 2, 4, 8, 16, …

   c) 1, -2, 4, -8, 16, …

   d) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

Deriving the Formula for the nth term of a Geometric Sequence

The general term or nth term in a geometric sequence is:

$$t_n = t_1 r^{n-1}$$

where
t_n is the nth term
t_1 is the first term
r is the common ratio
n is the number of terms

2. a) Determine the 10th term of the given geometric sequence.
   2, -6, 18, -54 …

b) Identify the sequence as convergent or divergent.

3. Given two terms in a finite geometric sequence are $t_1 = 7$, and $t_5 = 567$ and the last term of the sequence is 45 927, determine how many terms there are in the sequence.

4. Determine the value of x that would make $x, 2x + 2, 3x + 3$ a geometric sequence.

5. Insert two geometric means between -2 and 128.

6. A ball is dropped from a height of 3 m. After each bounce it rises to 75% of its previous height. Determine after how many bounces the ball will reach a height of approximately 40 cm.

7. In a geometric sequence the third term is 54 and the sixth term is -1458. Determine the values of t_1 and r.