Lesson 3 Geometric Sequences

A ***geometric sequence*** is formed by multiplying each term after the 1st term by a ***common ratio***, ***r***, to determine the next term.

The common ratio, *r*, is any non-zero real number that can be determined by dividing any term by the preceding term.

3, 6, 12, 24 … has a common ratio of 2

$r = \frac{t_2}{t_1}$ or $r = \frac{t_3}{t_2}$

3, 6, 12, 24 … is called an ***infinite geometric sequence***

3, 6, 12, 24 is called a ***finite geometric sequence***

Examples

1. Identify which of the following are geometric sequences.

a) 3, 6, 9, 12, …

not geometric (arithmetic where $d = 3$)

b) 2, 4, 8, 16, …

geometric $r = 2$

c) 1, -2, 4, -8, 16, …

geometric $r = -2$

d) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

geometric $r = \frac{1}{2}$

Deriving the Formula for the *n*th term of a Geometric Sequence

Let t_1 be the first term

∴ sequence formed

t_1

$t_2 = t_1 \cdot r$

$t_3 = t_2 \cdot r = (t_1 \cdot r) \cdot r = t_1 \cdot r^2$

$t_4 = t_3 \cdot r = (t_1 r^2) \cdot r = t_1 \cdot r^3$

⋮

$t_n = t_1 r^{n-1}$

The general term or *n*th term in a geometric sequence is:

$$t_n = t_1 r^{n-1}$$

where
t_n is the n^{th} term
t_1 is the first term
r is the common ratio
n is the number of terms

2. a) Determine the 10^{th} term of the given geometric sequence.

2, -6, 18, -54 …

$t_1 = 2$
$r = -3$
$n = 10$

$t_n = t_1 r^{n-1}$

$t_{10} = 2(-3)^{10-1}$

$= -39\,366$

b) Identify the sequence as convergent or divergent.

approaching the same sum as terms get closer and closer to 0

terms approach either $-\infty$ or ∞

divergent

terms are approaching $-\infty$ or ∞ alternating

3. Given two terms in a finite geometric sequence are $t_1 = 7$, and $t_5 = 567$ and the last term of the sequence is 45 927, determine how many terms there are in the sequence.

① Determine value of r.

$t_1 = 7$, $t_5 = 567$, $n = 5$

$t_n = t_1 r^{n-1}$

$567 = 7r^{5-1}$

$81 = r^4$

$\pm 3 = r$

② Determine value of n

$t_n = t_1 r^{n-1}$

$t_n = 45927$, $t_1 = 7$, $r = \pm 3$, $n = ?$

$45\,927 = 7(\pm 3)^{n-1}$

$6561 = (\pm 3)^{n-1}$

If the bases are equal, the exponents are equal

$(\pm 3)^8 = (\pm 3)^{n-1}$

$\therefore 8 = n-1$

$9 = n$

determine a common base

$3^8 = 6561$

$\therefore$ There are 9 terms in the sequence.

4. Determine the value of x that would make $x, 2x + 2, 3x + 3$ a geometric sequence.

$t_1 \quad t_2 \quad t_3$

$r = \frac{t_2}{t_1}$ or $r = \frac{t_3}{t_2}$

$\therefore \frac{t_2}{t_1} = \frac{t_3}{t_2}$

$\frac{2x+2}{x} = \frac{3x+3}{2x+2}$

$\frac{t_2}{t_1} \quad \frac{2x+2}{x} = \frac{3(x+1)}{2(x+1)}$

$r = \frac{t_2}{t_1}$ $\therefore r = \frac{3}{2}$

$x \neq -1, 0$

$4x + 4 = 3x$

$x = -4$

5. Insert two geometric means between -2 and 128.

$-2, \underline{8}, \underline{-32}, 128$

$t_1 \qquad t_4$

$t_4 = t_1 \cdot r^3$

$128 = -2r^3$

$-64 = r^3$

$-4 = r$

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2 c, g

3 b

4 c, g, h

7, 8

Try 5

6. A ball is dropped from a height of 3 m. After each bounce it rises to 75% of its previous height. Determine after how many bounces the ball will reach a height of approximately 40 cm.

$t_1 = 2.25m$

$t_n \approx 0.4m$

$t_n = t_1 r^{n-1}$

$0.4 = 2.25(0.75)^{n-1}$

$0.17 = 0.75^{n-1}$

$0.75^6 = 0.75^{n-1}$ $\quad 6 = n-1$

$\therefore n = 7$

$\therefore$ 7 bounces to reach approx 0.4 m or 40 cm

7. In a geometric sequence the third term is 54 and the sixth term is -1458. Determine the values of t_1 and r. t_3 t_6

$t_6 = t_3 r^3$

$-1458 = 54 r^3$

$-27 = r^3$

$-3 = r$

$54 = t_1(-3)^2$

$54 = 9t_1$

$6 = t_1$

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#4 i, j
10, 15, 18