

Lesson 3 Geometric Sequences

A *geometric sequence* is formed by multiplying each term after the 1st term by a common ratio, r , to determine the next term.

The common ratio, r , is any non-zero real number that can be determined by dividing any term by the preceding term.

3, 6, 12, 24 ... has a common ratio of 2

3, 6, 12, 24 ... is called an *infinite geometric sequence*

$$r = \frac{t_2}{t_1} \quad \text{or} \quad r = \frac{t_3}{t_2}$$

3, 6, 12, 24 ... is called a *finite geometric sequence*

Examples

1. Identify which of the following are geometric sequences.

a) 3, 6, 9, 12, ...

not geometric (arithmetic where $d=3$)

b) 2, 4, 8, 16, ...

geometric $r=2$

c) 1, -2, 4, -8, 16, ...

geometric $r=-2$

d) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$

geometric $r = \frac{1}{2}$

Deriving the Formula for the n th term of a Geometric Sequence

Let t_1 be the first term
 \therefore sequence formed
 t_1
 $t_2 = t_1 \cdot r$
 $t_3 = t_2 \cdot r = (t_1 \cdot r) \cdot r = t_1 \cdot r^2$
 $t_4 = t_3 \cdot r = (t_1 \cdot r^2) \cdot r = t_1 \cdot r^3$
 \vdots
 $t_n = t_1 r^{n-1}$

The general term or n th term in a geometric sequence is:

$$t_n = t_1 r^{n-1}$$

where
 t_n is the n^{th} term
 t_1 is the first term
 r is the common ratio
 n is the number of terms

2. a) Determine the 10th term of the given geometric sequence.
 2, -6, 18, -54 ...

$t_1 = 2$
 $r = -3$
 $n = 10$
 $t_n = t_1 r^{n-1}$
 $t_{10} = 2(-3)^{10-1}$
 $= -39\ 366$

approaching the same sum as terms get closer and closer to 0

- b) Identify the sequence as convergent or divergent.

divergent
 terms are approaching $-\infty$ or ∞
 alternating

terms approach either $-\infty$ or ∞

3. Given two terms in a finite geometric sequence are $t_1 = 7$, and $t_5 = 567$ and the last term of the sequence is 45 927, determine how many terms there are in the sequence.

① Determine value of r .
 $t_1 = 7$
 $t_5 = 567$
 $n = 5$
 $t_n = t_1 r^{n-1}$
 $567 = 7r^{5-1}$
 $81 = r^4$
 $\pm 3 = r$

② Determine value of n
 $t_n = 45927$
 $t_1 = 7$
 $r = \pm 3$
 $n = ?$
 $t_n = t_1 r^{n-1}$
 $45927 = 7(\pm 3)^{n-1}$
 $6561 = (\pm 3)^{n-1}$
 $(\pm 3)^8 = (\pm 3)^{n-1}$
 IF the bases are equal, the exponents are equal $\rightarrow \therefore 8 = n-1$
 $9 = n$
 determine a common base
 $3^8 = 6561$
 \therefore there are 9 terms in the sequence.

4. Determine the value of x that would make $x, 2x + 2, 3x + 3$ a geometric sequence.

$t_1 \quad t_2 \quad t_3$
 $r = \frac{t_2}{t_1}$
 or $r = \frac{t_3}{t_2}$
 $\therefore \frac{t_2}{t_1} = \frac{t_3}{t_2}$

$\frac{2x+2}{x} = \frac{3x+3}{2x+2}$
 $\frac{2x+2}{x} = \frac{3(x+1)}{2(x+1)}$
 $r = \frac{t_2}{t_1} \therefore r = \frac{3}{2}$

$x \neq -1, 0$
 $4x+4 = 3x$
 $x = -4$

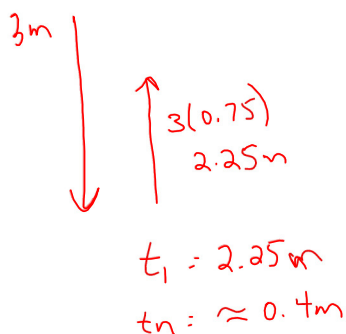
5. Insert two geometric means between -2 and 128.

$-2, \underline{8}, \underline{-32}, 128$
 $t_1 \quad \quad \quad t_4$

$t_4 = t_1 \cdot r^3$
 $128 = -2r^3$
 $-64 = r^3$
 $-4 = r$

pg. 358
 # 1c, h, i
 2c, g
 3b
 4c, g, h
 7, 8
 Try 5

6. A ball is dropped from a height of 3 m. After each bounce it rises to 75% of its previous height. Determine after how many bounces the ball will reach a height of approximately 40 cm.



$$t_n = t_1 r^{n-1}$$

$$0.4 = 2.25(0.75)^{n-1}$$

$$0.17 = 0.75^{n-1}$$

$$0.75^6 = 0.75^{n-1}$$

$$6 = n-1$$

$$\therefore n = 7$$

\therefore 7 bounces to reach approx 0.4m or 40cm

7. In a geometric sequence the third term is 54 and the sixth term is -1458. Determine the values of t_1 and r .

$$t_6 = t_3 r^3$$

$$-1458 = 54 r^3$$

$$-27 = r^3$$

$$-3 = r$$

$$54 = t_1 (-3)^2$$

$$54 = 9t_1$$

$$6 = t_1$$

pg 359
#4 i, j
10, 15, 18