$S_5 = 170$ $S_6 = 225$

4. The sum of the first 5 terms of an arithmetic series is 170. The sum of the first 6 terms is 225. The common difference is 7. Determine the first 4 terms of the series.

$S_5 = 170$
$S_6 = 225$ → 55
$d = 7$
$\therefore t_6 = 55$

$t_1 + t_2 + t_3 + t_4 + t_5 + t_6$
$170 + 55$
225
$\therefore t_6 = 55$

$S_n = \frac{n(t_1 + t_n)}{2}$

$225 = \frac{6(t_1 + 55)}{2}$

$225 = 3(t_1 + 55)$

$75 = t_1 + 55$

$20 = t_1$

$20 + 27 + 34 + 41 + \ldots$

5. Determine the sum of all multiples of 8 between 100 and 500.

first multiple of 8 is 104 ← t_1
last multiple of 8 is 496 ← t_n

$t_n = t_1 + d(n-1)$
$496 = 104 + 8(n-1)$
$392 = 8(n-1)$
$49 = n-1$
$50 = n$

$S_{50} = \frac{50(104 + 496)}{2}$

$= 15000$

Summation

If the summation expression is a linear function, then the summation is an arithmetic series.

ie. $\sum_{k=1}^{10} 3k+5$ is an arithmetic series since $f(x) = 3x + 5$ is linear.

↑ m

common difference

$\sum_{k=1}^{10} k^2+3$

is not an arithmetic series since $f(x) = k^2 + 3$ is not linear.

Examples

1. Evaluate.

$$\sum_{k=1}^{100} 2k+1$$

$t_1 = 2(1)+1$

$= 3$

$t_n = 2(100)+1$

$= 201$

$n = 100$

$S_{100} = \frac{100(3+201)}{2}$

$= 10\,200$

2. Express the given arithmetic series in summation notation.

$5 + 9 + 13 + \cdots + 137$

↑ t_1 ↑ t_n

$d = 4$

$t_n = t_1 + d(n-1)$

$137 = 5 + 4(n-1)$

$132 = 4(n-1)$

$33 = n-1$

$34 = n$

form $y = mx + b$

↑ slope is equal to d

$d = 4$

$t_1 = 4k + b$

$5 = 4(1) + b$

$1 = b$

$\therefore 4k+1$

$\sum_{k=1}^{34} 4k+1$

or

$\sum_{k=0}^{33} 4k+5$

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3a, c, d

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