Lesson 2 Arithmetic Series

A **series** is a sum of the terms in a sequence.
An **arithmetic series** is the **sum** of the terms in an **arithmetic sequence**.

For example:
1, 4, 7, 10 is an arithmetic sequence
$1 + 4 + 7 + 10$ is an arithmetic series
S_n is the partial sum of the first *n* terms of an arithmetic series.

The partial sum of *n* terms of an Arithmetic Series is given by

$$S_n = \frac{n(t_1 + t_n)}{2} \text{ or } S_n = \frac{n[2t_1 + d(n-1)]}{2}$$

where:
S_n is the partial sum of the first *n* terms
n is the number of terms
t_1 is the first term
d is the common difference
t_n is the n^{th} term

Examples

1. Determine the sum of the first 6 terms of the given arithmetic series.
   $25 + 14 + 3 - 8 - 19 - 30$

2. Given an arithmetic series has $t_1 = 3$ and $d = -4$, determine S_{25}.

3. An arithmetic series has $S_{32} = 1712, d = 3,$ and $t_{32} = 100$. Determine the first 3 terms of the series.

4. The sum of the first 5 terms of an arithmetic series is 170. The sum of the first 6 terms is 225. The common difference is 7. Determine the first 4 terms of the series.

5. Determine the sum of all multiples of 8 between 100 and 500.

Summation

If the summation expression is a linear function, then the summation is an arithmetic series.

ie. $\sum_{k=1}^{10} 3k + 5$ is an arithmetic series since $f(x) = 3x + 5$ is linear.

is $\sum_{k=1}^{10} k^2 + 3$ not an arithmetic series since $f(x) = k^2 + 3$ is not linear.

Examples

1. Evaluate.

$$\sum_{k=1}^{100} 2k + 1$$

2. Express the given arithmetic series in summation notation.

$5 + 9 + 13 + \cdots + 137$