

Lesson 2 Arithmetic Series

A **series** is a sum of the terms in a sequence.

An **arithmetic series** is the sum of the terms in an arithmetic sequence.

An arithmetic sequence is: 1, 4, 7, 10, ...

An arithmetic series is: $1 + 4 + 7 + 10 + \dots$

S_n , is used to represent the sum of the first n terms of a series.

The partial sum of n terms of an Arithmetic Series

For an arithmetic series with 1st term, t_1 , common difference, d , and n^{th} term, t_n , the partial sum of the first n terms, S_n , is:

$$S_n = \frac{n(t_1 + t_n)}{2} \quad \text{or} \quad S_n = \frac{n[2t_1 + d(n - 1)]}{2}$$

Examples

1. Determine the sum of the first 6 terms of this arithmetic series:

$$25 + 14 + 3 - 8 - 19 - 30 \dots$$

$$\begin{array}{cccccc} t_1 & t_2 & & & & t_6 \\ 25 & 14 & 3 & -8 & -19 & -30 \\ 39 & 42 & 34 & 15 & -15 & \end{array} \quad S_6 = -15$$

$$\text{or} \quad S_n = \frac{n(t_1 + t_n)}{2}$$

$$S_6 = \frac{6(25 - 30)}{2}$$

$$= -15$$

2. If an arithmetic series has $t_1 = 3$ and $d = -4$, determine S_{25} .

$$S_n = \frac{n(2t_1 + d(n-1))}{2}$$

$$S_{25} = \frac{25(2(3) + (-4)(25-1))}{2}$$

$$= \frac{25(-90)}{2}$$

$$= -1125$$

3. An arithmetic series has $S_{32} = 1712$, $d = 3$, and $t_{32} = 100$. Determine the first 3 terms of the series.

$$S_n = \frac{n(t_1 + t_n)}{2}$$

$$1712 = \frac{32(t_1 + 100)}{2}$$

$$1712 = 16(t_1 + 100)$$

$$107 = t_1 + 100$$

$$7 = t_1$$

$$\therefore t_1 = 7, t_2 = 10, t_3 = 13$$

Series $7 + 10 + 13 + \dots$

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4. The bottom row in a trapezoid had 49 cans. Each consecutive row had 4 fewer cans than the previous row. There were 11 rows in the trapezoid. Determine how many cans were in the trapezoid.

$$\begin{aligned}
 t_1 &= 49 \\
 d &= -4 \\
 n &= 11 \\
 S_n &= \frac{n(2t_1 + d(n-1))}{2} \\
 S_{11} &= \frac{11(2(49) + (-4)(11-1))}{2} \\
 &= \frac{11(58)}{2} \\
 &= 319
 \end{aligned}$$

\therefore there were 319 cans

5. The sum of the first 5 terms of an arithmetic series is 170. The sum of the first 6 terms is 225. The common difference is 7. Determine the first 4 terms of the series.

$$\begin{aligned}
 S_5 &= 170 \\
 S_6 &= 225 \\
 d &= 7
 \end{aligned}
 \begin{array}{l}
 \text{add} \\
 55 \\
 \uparrow \\
 t_6
 \end{array}
 \begin{aligned}
 S_n &= \frac{n(t_1 + t_n)}{2} \\
 225 &= \frac{6(t_1 + 55)}{2} \\
 225 &= 3(t_1 + 55) \\
 75 &= t_1 + 55 \\
 20 &= t_1
 \end{aligned}$$

$$\therefore 20 + 27 + 34 + 41$$

Exercise 2 Arithmetic Series

- 1.) Determine the sum of the first 6 terms of this arithmetic series:
 $15 + 11 + 7 + 3 - 1 - 5 \dots$ (follow example 1) 30
- 2.) If an arithmetic series has $t_1 = 5$ and $d = 4$, determine S_{20} . (follow example 2) 860
- 3.) If an arithmetic series has $t_1 = 7$, $t_n = 79$ and $n = 8$, determine S_n . 344
- 4.) An arithmetic series has $S_{40} = 4940$, $d = -3$, and $t_{40} = 65$. Determine the first 3 terms of the series. (follow example 3) $182, 179, 176$
- 5.) An auditorium has 8 seats in the first row. Each subsequent row has four more seats than the previous row. Determine how many seats are in the 50th row of the auditorium. (follow example 4) 204
- 6.) Determine how many terms of the arithmetic series $1491 + 1484 + 1477 + \dots$ are needed to give a sum of 0. 427
- 7.) Determine the value of n for an arithmetic series where $t_1 = 8$, $t_n = 68$ and $S_n = 608$. 16
- 8.) The sum of the first five terms of an arithmetic series is 85. The sum of the first six terms is 123. Determine the first four terms of the series.
 $3, 10, 17, 24$

Extra practice: Pg. 19; #3a, d, 4a, 5a, 6a, 7a,c, 9, 10a, 12, 13