Lesson 1 Arithmetic Sequences

Definition:
A sequence is a list of numbers where each number is called a term of the sequence. The notation is often $t_1, t_2, t_3 \ldots t_n$ where t_n is the last term in a finite sequence. If the terms continue forever such as $t_1, t_2, t_3 \ldots$ then the sequence is considered an infinite sequence.

An ***arithmetic sequence*** is a sequence in which each term after the first is obtained by adding or subtracting a ***common difference***, ***d***, to the previous term.

For example: t_1 t_2 t_3 t_4
For the sequence, 4, 7, 10, 13 $d = 3$
If the first term of this sequence is: t_1,
then the second term is: $t_1 + 3$
the third term is: $t_2 + 3$
the fourth term is: $t_3 + 3$, and so on.

This can also be written as
$t_1 = 4$
$t_2 = 4 + 3(1) = 7$
$t_3 = 4 + 3(2) = 10$
$t_4 = 4 + 3(3) = 13$
We can use this pattern to come up with the general form of an arithmetic sequence that can be used to determine any term of the sequence without having to list all of the numbers in the sequence.

General Form of an Arithmetic Sequence
$t_n = t_1 + d(n-1)$

where
t_1 is the first term
d is the common difference
n is the number of terms
t_n is the n^{th} term

Note:
4, 7, 10, 13 is considered an increasing, finite sequence.
5, 2, -1, -4… is considered a decreasing, infinite sequence.

Example

1. Given the arithmetic sequence $-3, 2, 7, 12, \ldots$
   a) determine t_{20}.
   b) determine which term in the sequence has the value 212.

some n^{th} term in the sequence $\therefore t_n = 212$

a) $t_n = t_1 + d(n-1)$

$t_n = -3 + 5(n-1)$ ← general term

$t_{20} = -3 + 5(20-1)$

$t_{20} = 92$

t_{20} ↑ n

$t_1 = -3$

$d = 5$

b) $t_n = t_1 + d(n-1)$

$212 = -3 + 5(n-1)$

$215 = 5(n-1)$

$43 = n-1$

$44 = n$

$t_1 = -3$
$d = 5$
$n = ?$
$t_n = 212$

212 is the 44^{th} term

$\therefore t_{44} = 212$

2. Given the arithmetic sequence $3, 10, 17, 24, \ldots$
   a) determine t_{15}.
   b) determine which term in the sequence has the value 220.

a) $t_{15} = 3 + 7(15-1)$

$= 101$

b) $220 = 3 + 7(n-1)$

$217 = 7(n-1)$

$31 = n-1$

$32 = n$

$\therefore t_{32} = 220$

3. Given two terms in an arithmetic sequence are $t_4 = -4$, and $t_7 = 23$. Determine the value of t_1.

$\_, \_, \_, \underset{t_4}{-4}, \_, \_, \underset{t_7}{23}$ (d, d, d)

Note $7-4=3$

$t_7 = t_4 + 3d$

$23 = -4 + 3d$

$27 = 3d$

$9 = d$

$t_1 = t_4 - 3d$
$= -4 - 3(9)$
$= -31$

or

$t_n = t_1 + d(n-1)$
$-4 = t_1 + 9(4-1)$
$-4 = t_1 + 27$
$-31 = t_1$

same as example 1

4. Given two terms in an arithmetic sequence are $t_4 = -4$, and $t_7 = 23$. Determine the value of t_{11}.

$t_{11} = t_1 + d(11-1)$
$= -31 + 9(11-1)$
$= 59$

5. Determine the value of x so that $x + 3$, $2x + 1$, and $5x + 2$ are consecutive terms of an arithmetic sequence.

$d = \underbrace{2x+1}_{t_2} - \underbrace{(x+3)}_{t_1}$

or

$d = \underbrace{5x+2}_{t_3} - \underbrace{(2x+1)}_{t_2}$

$$2x+1-(x+3) = 5x+2-(2x+1)$$
$$2x+1-x-3 = 5x+2-2x-1$$
$$x-2 = 3x+1$$
$$-3 = 2x$$
$$-\frac{3}{2} = x$$

sometimes t_1 is denoted as "a"

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\# 4a, 7e
8 c
9 a, e
10 d, f
11 b, d
15

4a) $a = 4$ $(t_1 = 4)$

$t_n = 2 + t_{n-1}$

$t_2 = 2 + \underbrace{t_{2-1}}_{t_1}$

$t_2 = 2 + 4$

$t_2 = 6$

4, 6, $\underline{8}$, $\underline{10}$

$t_3 = 2 + \underbrace{t_{3-1}}_{t_2}$

$= 2 + 6$