Lesson 4 Logs with Special Bases

The most important base number for exponential functions is the value denoted by e.
Like π, e (euler's number) is an irrational number whose value is 2.718281828347045…

Evaluating Logarithms

Base 10

- $y = \log_{10} x$ is called a common logarithmic function
- also written as $y = \log x$

↑ base 10 is implied

Base e

- the inverse of the natural exponential function $y = e^x$ is $y = \log_e x$ which is more commonly written as $y = \ln x$

$x = e^y$

* pronounced "lon" x

Ex.1) Use your calculator to find the following logs.

* check to make sure know how to use your calc

a) log 100 — 2

b) $\log\left(\frac{1}{10}\right)$ — −1

c) ln 100 — 4.605

d) $\ln e^3$ — 3

e) ln 1 — 0

f) $\ln e^7$ — 7

g) log 0 — ∅ (argument must be positive)

h) log 1 — 0

i) ln e — 1

Ex. 2) Use your calculator to find the inverse logarithm (ie. Solve for x)

a) $\log x = 5$ — 100000

on calc: 2nd 10^x log 5

b) $ln_e x = -0.3$ — 0.741

on calc: 2nd e^x ln (-0.3)

*The calculator is only useful for base 10 and base e (special bases). For non-special bases we use exponential form or change of base formula.

Change of Base Theorem

$$\log_b n = \frac{\log_a n}{\log_a b} \quad \text{where } a, b \text{ and } n > 0, a \neq 1, b \neq 1$$

* allows you to change to base 10 or e so you can use the calc.

Ex. 3) Evaluate the following

a) $y = \log_2 3$

$\log_2 3 = \frac{\log_{10} 3}{\log_{10} 2} = \frac{\log 3}{\log 2}$

$= 1.585$

} don't need to write base as 10 is implied

b) $y = \log_7 \pi$

$\log_7 \pi = \frac{\log \pi}{\log 7}$

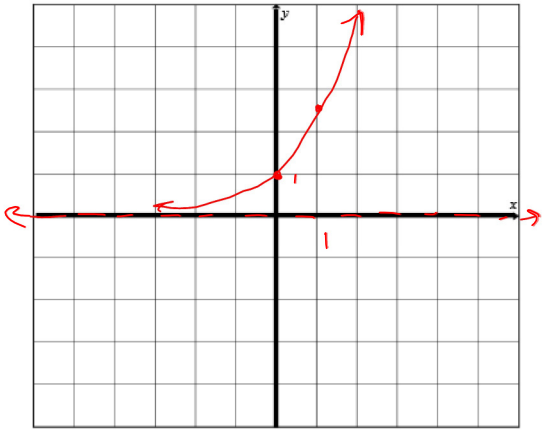
$= 0.588$

Sketching $y = e^x$ and $y = \ln x$

Ex. 1) Sketch the graph of $y = e^x$.

$e = 2.718$ ↗ Euler's number

x	y
0	1
1	2.718



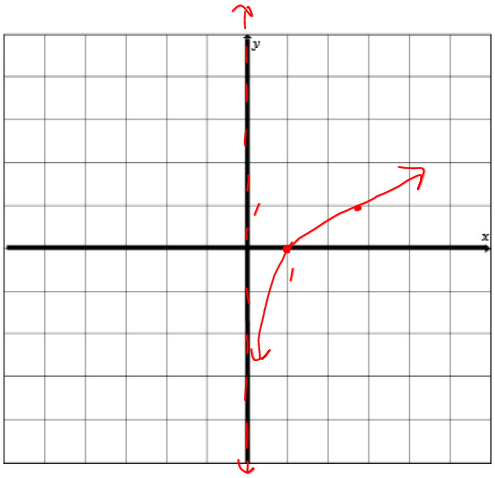
Ex. 2) Sketch the graph of $y = \ln x$.

- The inverse of $y = e^x$ *is* $y = \log_e x$
- More commonly written as $y = \ln x$

* switch x and y values

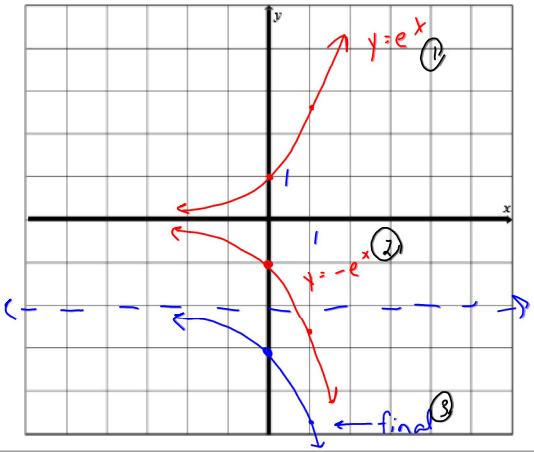
$y = e^x$
$x = e^y$

x	y
1	0
2.718	1



Ex. 3) Sketch the graph of $y = -e^x - 2$.

↑ reflection over the x-axis ↑ down 2



Assignment: Sketch the graph of: 1.) $y = -e^{-x+1}$, 2.) $y = \ln(-x) + 1$, 3.) $y = 2e^{(x-1)}$, 4.) $y = -\ln(-x + 2)$ change of base sheet (odds)