Pre-Calculus 12 Solving Trig Equations Algebraically (Substitution and Double Angles)

Ex. 1) Solve: $\sqrt{2}\csc\theta = -5$ *for* $-180° \le \theta \le 180°$

degrees mode

Recall:
$\csc\theta = \frac{1}{\sin\theta}$
$\sec\theta = \frac{1}{\cos\theta}$
$\cot\theta = \frac{1}{\tan\theta}$

① Isolate csc θ

② take the reciprocal

③ Find θr

$\csc\theta = \frac{-5}{\sqrt{2}}$

$\sin\theta = -\frac{\sqrt{2}}{5}$

$\theta_r = \sin^{-1}\left(\frac{\sqrt{2}}{5}\right)$

$\theta_r = 16.4299...°$ ← not a sol'n

④ Find values in Quadrants where $\sin\theta < 0$

$\sin\theta < 0$ in Q III, IV

QIII: $\theta = 180° + 16.4299...°$
$= 196.430°$

QIV $\theta = 360° - 16.4299...°$
$= 343.570°$

⑤ not in interval, find coterminal angles

$\theta_c = 196.430° - 360°$
$= -163.570°$

$\theta_c = 343.570° - 360°$
$= -16.430°$

$\theta = -163.570°, -16.430°$

Ex. 2) Solve: $2\cos^2\theta - 3\sin\theta = 0$ *for* $0 \le \theta \le 2\pi$

2 diff trig fcns

① Use substitution to change to one

Recall

$\sin^2\theta + \cos^2\theta = 1$ identity

$\cos^2\theta = 1 - \sin^2\theta$

$2(1 - \sin^2\theta) - 3\sin\theta = 0$

② Distribute 2 through

$2 - 2\sin^2\theta - 3\sin\theta = 0$

$0 = 2\sin^2\theta + 3\sin\theta - 2$

③ Factor

$0 = (2\sin\theta - 1)(\sin\theta + 2)$

p -4
s 3
F 4 -1
2 1

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\sin\theta = -2$ rej No sol'n

Ex. 3) Solve for θ: $4\sin^2\theta - 7\cos\theta - 2 = 0$ *for* $0 \le \theta \le 2\pi$

$4(1-\cos^2\theta) - 7\cos\theta - 2 = 0$

$4 - 4\cos^2\theta - 7\cos\theta - 2 = 0$

$0 = 4\cos^2\theta + 7\cos\theta - 2$

$0 = (4\cos\theta - 1)(\cos\theta + 2)$

$\cos\theta = \frac{1}{4}$ $\quad \cos\theta \neq -2$ No sol'n

$\theta_r = \cos^{-1}\left(\frac{1}{4}\right)$

$\theta_r = 1.31812\ldots$

$\cos\theta > 0$ in QI, QIV

QI $\theta = 1.318$

QIV $\theta = 2\pi - 1.31812\ldots$

$= 4.965$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

Also on formula sheet

$1 + \cot^2\theta = \csc^2\theta$

$\tan^2\theta + 1 = \sec^2\theta$

Double Angles

Ex. 4) Solve $\cos(2\theta) = 1$ *for* $0 \le \theta \le 2\pi$

Replace $\cos 2\theta$ with a double angle identity

$1 - 2\sin^2\theta = 1$

$0 = 2\sin^2\theta$

$0 = \sin^2\theta$

$0 = \sin\theta$

$\theta = 0, \pi, 2\pi$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$ don't use often

$\cos 2\theta = 1 - 2\sin^2\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

Recall:

If $\sin^2\theta = 1$,

then $\sin\theta = \pm 1$

Ex. 5) Solve $2\sin^2 x = \cos 2x \quad for \ \frac{\pi}{2} \le x \le \frac{3\pi}{2}$

$2\sin^2 x = 1 - 2\sin^2 x$ (same trig fcn)

$\cos 2x = 1 - 2\sin^2 x$

Use substitution to replace double angle

Combine like terms

Isolate $\sin^2 x$

$4\sin^2 x = 1$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm \frac{1}{2}$

Quads II & III

$\frac{\pi}{2} \le x \le \frac{3\pi}{2}$

$\frac{3\pi}{6} \le x \le \frac{9\pi}{6}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

not in given interval

$\therefore x = \frac{5\pi}{6}, \frac{7\pi}{6}$

worksheet #1, 2, 3c 5, 6a-c

Assignment: Handout "Trig Equations Worksheet" #1a, 2c, 3b,c, 4a, 5a,c,e, 6b