Pre-Calculus 11 Solving Rational Equations

Solving equations is one of the most important aspects of algebra. Remember some of the techniques you've already learned, such as:

Recall:

- If linear, just isolate the variable and solve

$\frac{x}{4} + 3 = 7$

$\frac{x}{4} = 4$

$x = 16$

- $\frac{m}{3} + \frac{3m}{4} = 13$ $\times 12$

Multiply each term by 12

LCM 12

$4m + 3(3m) = 156$

$4m + 9m = 156$

$13m = 156$

$m = 12$

- If quadratic, set = 0, factor, set each factor = 0 and solve

Solve: $x^2 - 4x = 5$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x - 5 = 0 \quad x + 1 = 0$

$x = 5 \quad x = -1$

The same strategy can be used to solve Rational Equations.

Steps to Solve Rational Equations:

- Factor and State Restrictions
- Cross Multiply (if possible)
- Multiply by LCD (only numerator) to get rid of fractions
- Solve for variable
- Check solution is a permissible value, if not, it is an ***extraneous root***.

Examples: Solve each of the following

1. $\frac{x}{x+2} = \frac{5}{7}$ cross multiply

LCM $7(x+2)$

$7x = 5(x+2)$

$7x = 5x + 10$

$2x = 10$

$x = 5$

$x \neq -2$

2. $\frac{x+1}{x-1} = \frac{x-1}{x+3}$

LCM $(x-1)(x+3)$

$(x+1)(x+3) = (x-1)(x-1)$ $\quad x \neq -3, 1$

$x^2 + 4x + 3 = x^2 - 2x + 1$

$6x = -2$

$x = -\frac{1}{3}$

3. $\frac{3}{1} + \frac{1}{x} = \frac{4}{x^2}$ $\quad x \neq 0$

LCM x^2

$3x^2 + x = 4$

$3x^2 + x - 4 = 0$

P -12
S 1
F $\frac{4}{1}, \frac{-3}{3}$

$(x-1)(3x+4) = 0$

$x = 1 \quad x = -\frac{4}{3}$

4. $\frac{3x+1}{x^2-1} = \frac{-x}{x+1}$

$\frac{3x+1}{(x+1)(x-1)} = \frac{-x}{x+1}$ $\quad x \neq \pm 1$

LCM $(x+1)(x-1)$

$3x+1 = -x(x-1)$

$3x+1 = -x^2+x$

$x^2+2x+1 = 0$

$(x+1)(x+1) = 0$

$x = -1$ rej, extraneous root

$\therefore$ no sol'n

* Multiply each numerator by whatever is missing from the LCM in the denominator

5. $\frac{x}{x-3} = \frac{-6}{x^2-8x+15}$

$\frac{x}{x-3} = \frac{-6}{(x-3)(x-5)}$ $\quad x \neq 3, 5$

LCM $(x-3)(x-5)$

$x(x-5) = -6$

$x^2-5x = -6$

$x^2-5x+6 = 0$

$(x-3)(x-2) = 0$

$x = 3$ rej $\quad \boxed{x = 2}$

6. $\frac{9}{3x-6} = \frac{2}{x} + \frac{3}{x-2}$

$\frac{9}{3(x-2)} = \frac{2}{x} + \frac{3}{x-2}$ $x \neq 0, 2$

LCM $3x(x-2)$

$9x = 6(x-2) + 9x$

$9x = 6x - 12 + 9x$

$12 = 6x$

$2 = x$

$\therefore$ no sol'n

Review pg. 606

Assignment: Pg. 584; #5a, 6b, 7c, 8, 9a, 10a