Pre-Calculus 12 Solving Logarithmic Equations

Steps:

1. Move all logs on one side and leave the constant (or 0) on other side. If all terms have logs, no need to isolate.
2. Combine all logs into a single log using log laws.
3. Change to exponential form
4. Solve
5. Check your solution, extraneous root may exist
   - Logs are only defined for positive (+) arguments, if a solution yields a negative (-) argument, reject that solution.

recall: $y = \log_a x \Rightarrow a^y = x$ change to exponential form

exponent, argument

$$y = \log_b x$$

base

Ex. 1) Solve

a) $\log_3(2x) = \log_3(x+5)$

single log on each side, arguments must be equal (same idea as solving exponential eqns w/common bases)

$\therefore 2x = x + 5$

$x = 5$

check that $x = 5$ doesn't make either argument negative

arguments ie $2x$ and $x+5$ are the arguments

sub $x = 5$ in: $2(5)$ $\quad 5+5$

both are positive $\therefore$ $\boxed{x = 5}$

b) $\log_3(9x) + \log_3 x = 4$

$\log_3(9x^2) = 4$ Product law

$3^4 = 9x^2$ Exponential form

$81 = 9x^2$

$9 = x^2$

$\pm 3 = x$ careful

3^4

$3^2 \cdot 3^2$

$9 \cdot 9$

* check arguments

$x = 3$ $x = -3$ reject

* makes the argument(s) -ve

$9(-3) = -27$ or $\log(-3)$

$\log_3(-27)$

not possible argument can't be negative

c) $\log_5(3x+1) + \log_5(x-3) = 3$

$\log_5[(3x+1)(x-3)] = 3$ combine to a single log, using product law

$\log_5(3x^2 - 8x - 3) = 3$

$5^3 = 3x^2 - 8x - 3$ exponential form

$0 = 3x^2 - 8x - 128$

P -384

S -8

F $-24, 16$

$0 = (3x+16)(x-8)$

$x = -\frac{16}{3}$ rej $x = 8$

* check arguments

$3x^2 - 8x - 128 = 0$

or use the quadratic formula

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{8 \pm \sqrt{64 - 4(3)(-128)}}{2(3)}$

$= \frac{8 \pm \sqrt{1600}}{6}$

$= \frac{8 \pm 40}{6}$

$x = \frac{8+40}{6}$ or $x = \frac{8-40}{6}$

$= \frac{48}{6}$ $= -\frac{32}{6}$

$= 8$ $= -\frac{16}{3}$

d) $\log(6x) = \log(x+6) + \log(x-1)$

$\log(6x) = \log[(x+6)(x-1)]$

equate arguments

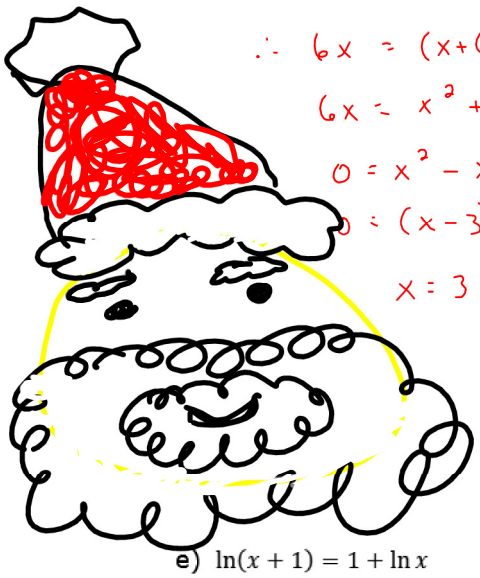
$\therefore 6x = (x+6)(x-1)$

$6x = x^2 + 5x - 6$

$0 = x^2 - x - 6$

$0 = (x-3)(x+2)$

$x = 3 \quad \cancel{x = -2}$ reject



e) $\ln(x+1) = 1 + \ln x$

$\ln(x+1) - \ln x = 1$

$\ln\left(\frac{x+1}{x}\right) = 1$ quotient law

$e^1 = \frac{x+1}{x}$ exponential form (base e)

$ex = x + 1$

GCF $ex - x = 1$

$x(e-1) = 1$

$x = \frac{1}{e-1}$

worksheet

b, c, d, h, j

k, n, o, p

b) $\frac{-6\log_3(x-3)}{-6} = \frac{-24}{-6}$

$\log_3(x-3) = 4$