Lesson 6 Solving Logarithmic Equations

Steps:

1. Move all logs to one side and leave the constant (or 0) on other side. If all terms have logs, no need to isolate (see example 1).
2. Combine all logs into a single log using log laws.
3. Change to exponential form
4. Solve
5. Check your solution, extraneous roots may exist
   - ➢ Logs are only defined for positive (+) arguments, if a solution yields a negative (-) or 0 argument, reject that solution.

Ex. 1) Solve.

a) $\log_3(2x) = \log_3(x+5)$

single log on each side

∴ arguments must be equal

(same idea as solving exponential eqns with common bases)

$2x = x + 5$

$x = 5$

* check to make sure $x = 5$ won't make either argument (2x or x+5) 0 or negative.

$2(5)$ and $5 + 5$ are both positive

∴ $x = 5$ is a valid solution

b) $\log_3(9x) + \log_3 x = 4$

$\log_3(9x^2) = 4$ Product law

$3^4 = 9x^2$ Exponential form

$81 = 9x^2$

$9 = x^2$

$\pm 3 = x$

$\therefore x = 3$ $x \neq -3$ makes the arguments negative

3^{-1}
3^0
3^1
} positive

c) $\log_5(3x + 1) + \log_5(x - 3) = 3$

$\log_5((3x+1)(x-3)) = 3$

$5^3 = 3x^2 - 8x - 3$ Put in exponential form

$0 = 3x^2 - 8x - 128$

$0 = (3x+16)(x-8)$

P -384
S -8
F $-\frac{24}{3}, \frac{16}{1}$

$x = \cancel{\frac{-16}{3}}$ rej $x = 8$

d) $\log(6x) = \log(x+6) + \log(x-1)$

$\log(6x) = \log((x+6)(x-1))$

Product law to combine logs

Single log on both sides, so equate arguments

$\therefore \quad 6x = x^2 + 5x - 6$

$0 = x^2 - x - 6$

$0 = (x-3)(x+2)$

$x = 3 \qquad x = -2$ rej

e) $\ln(x+1) = 1 + \ln x$

$\ln(x+1) - \ln x = 1$

$\ln\left(\frac{x+1}{x}\right) = 1$

$e^1 = \frac{x+1}{x}$

$ex = x + 1$

$ex - x = 1$

GCF $\quad x(e-1) = 1$

$x = \frac{1}{e-1}$ ← exact value

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Enrichment

f) $x^{\log x} = 100x$

change to log form $\log_x 100x = \log x$

change of base $\dfrac{\log 100x}{\log x} = \log x$

mult by $\log x$ $\log 100x = (\log x)^2$

Product law $\log 100 + \log x = (\log x)^2$

evaluate $\log 100$ $2 + \log x = (\log x)^2$

$0 = (\log x)^2 - \log x - 2$ ← trinomial pattern

P -2
S -1
F -2, 1

factor $0 = (\log x - 2)(\log x + 1)$

solve

$\log x = 2$ | $\log x = -1$

exp form $10^2 = x$ | $10^{-1} = x$

$100 = x$ | $\frac{1}{10} = x$

$3^2 = 9$

$\log_3 9 = 2$

$\log_3 9 = \dfrac{\log 9}{\log 3}$