Pre-Calculus 12
Solving Exponential Equations (with common bases)

Review Exponent Laws

$x^n \cdot x^m = x^{n+m}$

$\frac{x^n}{x^m} = x^{n-m}$

$(x^n)^m = x^{nm}$

$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

$x^0 = 1$

$x^{-n} = \frac{1}{x^n}$

$x^{n/m} = \sqrt[m]{x^n}$

ie $x^{\frac{1}{2}} = \sqrt{x}$

$x^{\frac{2}{3}} = \sqrt[3]{x^2}$

Steps to solve an exponential equation:

1. If the bases are the same (one base on each side) use one-to-one property;
   - equate the exponents and solve

   If $b^m = b^n$, then $m = n$
2. If bases are different;
   - rewrite with the same base
   - equate the exponents and solve

Ex. 1) Solve for x.

a) $2^{5x-1} = 16$

$2^{5x-1} = 2^4$

$\therefore 5x - 1 = 4$

$5x = 5$

$x = 1$

← Since 2^4 can only equal 2^4, we can equate exponents and solve for x

b) $4^{x+2} = 64^x$

$4^{x+2} = (4^3)^x$

$4^{x+2} = 4^{3x}$

$\therefore x + 2 = 3x$

$2 = 2x$

$1 = x$

Recall: $2^{-3} = \frac{1}{2^3}$

c) $3^x(27) = 81^{2x+1}$

$3^x \cdot 3^3 = (3^4)^{(2x+1)}$

$3^{x+3} = 3^{8x+4}$

$\therefore\ x+3 = 8x+4$

$-1 = 7x$

$-\frac{1}{7} = x$

* need one base on each side, use exp laws

d) $2^{3x} \cdot 4^{x-1} = \left(\frac{1}{8}\right)^{x+2}$

$2^{3x} \cdot (2^2)^{(x-1)} = (2^{-3})^{(x+2)}$ — mult exps

add exp → $2^{3x} \cdot 2^{2x-2} = 2^{-3x-6}$

$2^{5x-2} = 2^{-3x-6}$ ← one base each side, so now we can equate exps.

$\therefore\ 5x-2 = -3x-6$

$8x = -4$

$x = -\frac{1}{2}$

e) $4^{x+1} = 2^x\sqrt{2}$

$(2^2)^{(x+1)} = 2^x \cdot 2^{\frac{1}{2}}$

$2^{2x+2} = 2^{x+\frac{1}{2}}$

$\therefore\ 2x+2 = x+\frac{1}{2}$

$x = -\frac{3}{2}$

or

$4x+4 = 2x+1$

$2x = -3$

$x = -\frac{3}{2}$

f) $\left(\frac{1}{2}\right)^{-x^2} = 8^{2x-3}$

$(2^{-1})^{-x^2} = (2^3)^{(2x-3)}$

$2^{x^2} = 2^{6x-9}$

$\therefore\ x^2 = 6x-9$

$x^2 - 6x + 9 = 0$

$(x-3)(x-3) = 0$

$x = 3$

To solve for a missing base, raise both sides of the equation to the reciprocal power of the given exponent.

Ex. 2) Solve.

a.) $b^4 = 16$

$\sqrt[4]{b} = \sqrt[4]{16}$

$b = \pm 2$

b.) $b^{\frac{2}{3}} = 9$

$\left(b^{\frac{2}{3}}\right)^{\frac{3}{2}} = 9^{\frac{3}{2}}$ ← raise both sides to the reciprocal power

$b = (\sqrt{9})^3$

$b = 27$

Recall: Anything times its reciprocal will equal 1

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\# 1b, c

2c, 3c

4b, d

5a, c, d

Create Connections C2 (pg. 365)