Pre-Calculus 11 Solving Absolute Value Equations

Solving an Absolute value Equation graphically

Solve by graphing, then verify the solution.

1. $2 = |x - 4|$

To find a solution, find the x-values where $y = |x - 4|$, and $y = 2$ meet.

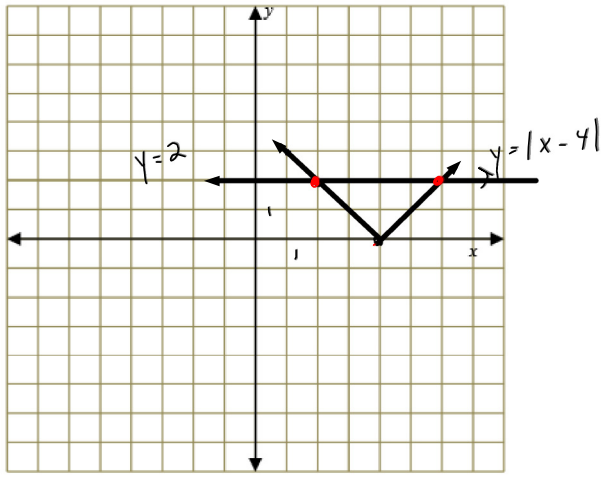
horizontal line

① Graph $y = x - 4$ or V-shape translated 4 units right

② Reflect negative y-values

only x-values

Sol'n $x = 2, 6$



checks

$2 = |2 - 4|$
$2 = |-2|$
$2 = 2$ ✓

$2 = |6 - 4|$
$2 = |2|$
$2 = 2$ ✓

For which values of c does the equation $|x - 4| = c$ have one solution? No solution?

$|x - 4| = 0$

$x - 4 = 0$

$x = 4$

only one value of x will make the LHS = 0

$c = 0$

$c < 0$

ie

$|x - 4| \neq -3$

every absolute value is positive ∴ will never be equal to -3

$|1 - 4| = -3$

$|-3| = -3$

$3 \neq -3$

2. $|x^2 + 8x + 16| = 1$

$y = |x^2 + 8x + 16|$

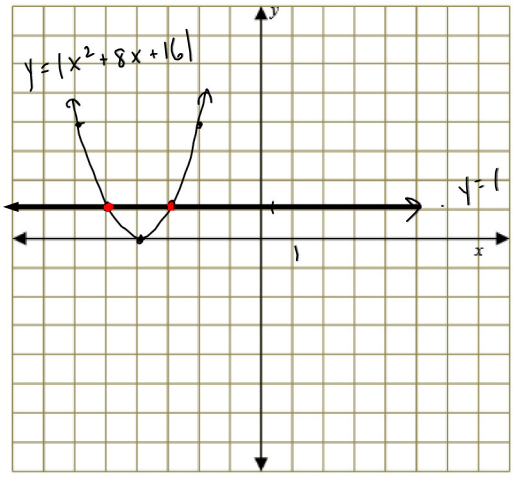
$y = |(x+4)^2|$

No negative values of y to reflect

$y = 1$

$x = -3, -5$

* Eqn only has x, only give x-values



An absolute value equation for the form $|ax^2 + bx + c| = d$ can have 0, 1, 2, 3, or 4 solutions. The number of solutions depends on the absolute value function graphed and the value of d.

See page 634 for supporting diagrams

Solving Absolute Value Equations Algebraically

1. $|3x + 1| = 7$

$3x+1 = 7$ or $-(3x+1) = 7$

$3x = 6$ $3x+1 = -7$

$x = 2$ $3x = -8$

$x = -\frac{8}{3}$

check sub into original eqn w/ absolute value

$|3(2)+1| = 7$

$|7| = 7$

$7 = 7$ ✓

$|\cancel{3}(-\frac{8}{\cancel{3}}) + 1| = 7$

$|-8+1| = 7$

$|-7| = 7$

$7 = 7$ ✓

* Solutions must be verified for full marks

2. $2|2x - 1| = 4x$

$\div 2$ $|2x-1| = 2x$

* Always isolate the abs value first!

$2x-1 = 2x$ or $-(2x-1) = 2x$

$-1 \neq 0$ $2x-1 = -2x$

no values of x $4x = 1$

$x = \frac{1}{4}$

check $2|2(\frac{1}{4}) - 1| = 4(\frac{1}{4})$

$2|\frac{1}{2} - 1| = 1$

$2|-\frac{1}{2}| = 1$

$2(\frac{1}{2}) = 1$

$1 = 1$ ✓

3. $|x^2 - 3x| = -4x + 6$

$x^2 - 3x = -4x + 6$ or $\frac{-(x^2 - 3x)}{-1} = \frac{-4x + 6}{-1}$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3$ $\quad$ ~~$x = 2$~~ rej

$x^2 - 3x = 4x - 6$

$x^2 - 7x + 6 = 0$

$(x-6)(x-1) = 0$

~~$x = 6$~~ rej $\quad$ $x = 1$

check

$x = -3$

$|(-3)^2 - 3(-3)| = -4(-3) + 6$

$|18| = 18$

$18 = 18$ ✓

$x = 2$

$|2^2 - 3(2)| \neq -4(2) + 6$

$|-2| \neq -2$

$2 \neq -2$

$x = 6$

$|6^2 - 3(6)| \neq -4(6) + 6$

$|18| \neq -18$

$18 \neq -18$

$x = 1$

$|1^2 - 3(1)| = -4(1) + 6$

$|-2| = 2$

$2 = 2$ ✓

Review pg 692
Practice Test pg. 698
or
extra practice booklet

Assignment: Pg. 639; #6a, 7a, 8a,c, 9, 11a,b