Lesson 4 Polynomial Functions

The degree of a polynomial function is the highest power of the variable in the equation.

Polynomial Functions

A polynomial function of degree n can be written in standard form as:

$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0$ where n is a non-negative integer.

The coefficient of the highest power of x is the leading coefficient.

The graph of a polynomial function is smooth and continuous, which means it has no sharp corners and can be drawn without lifting the pencil from the paper.

Sketching Polynomials

Steps:

1. Plot y-intercept (let $x = 0$)
2. Plot zeros (x-intercepts)
3. Check the coefficient and power of the leading term (positive or negative; odd or even)

Note: When in doubt, you can always use a table of values to find points in between to help with sketching.

* Make sure you have read through Polynomial Functions notes booklet first.

Ex. 1) Sketch the following: $f(x) = 1(x+2)(x-1)(x-3)$

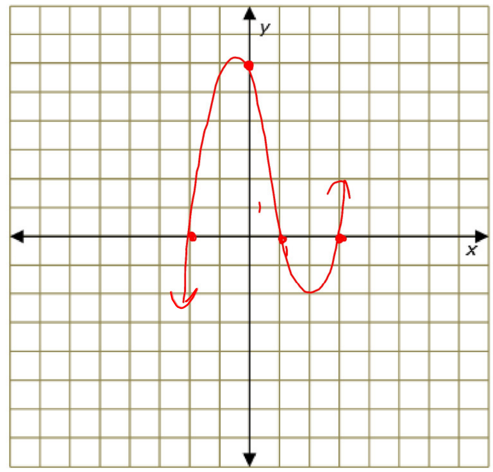
cubic function

y-int
$f(0) = (0+2)(0-1)(0-3)$
$= 6$

x-int
$(x+2)(x-1)(x-3) = 0$
$x+2=0 \quad x-1=0 \quad x-3=0$
$x=-2 \quad x=1 \quad x=3$

degree is 3 ← odd
opposite end behaviour

leading coeff 1
$a > 0$
fall left
rise right (ends positive)



Ex. 2) Sketch the following: $y = -x(x+4)(x-2)(x-5)$

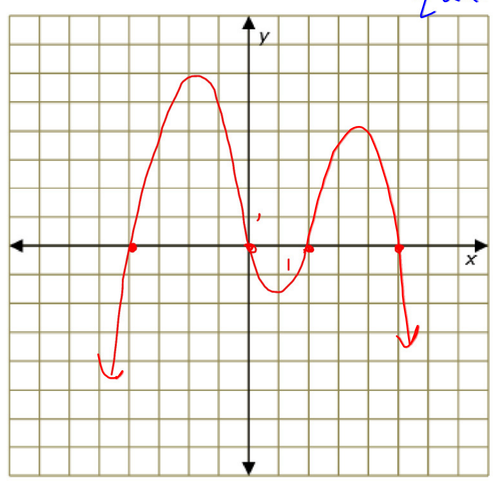
quartic function

y-int
$y = -0(0+4)(0-2)(0-5)$
$= 0$

x-int
$x = -4, 0, 2, 5$

degree 4 ← even
same end behaviour

leading coeff -1
$a < 0$, open down
falls left
falls right



Ex. 3) Sketch the graph: $g(x) = (x+3)(x+1)(x-1)^2$ ← multiplicity of 2

bounce

y-int

$g(0) = (0+3)(0+1)(0-1)^2$

$= 3$

x-int

$x = -3, \pm 1$

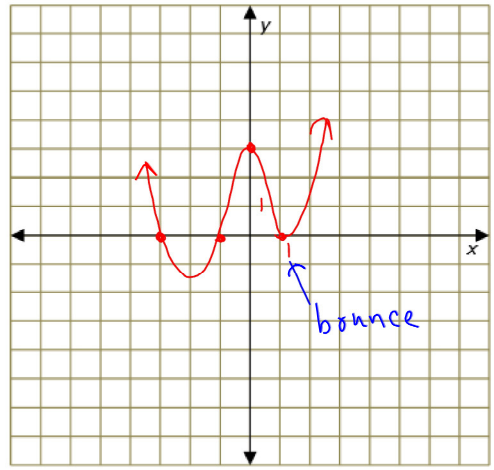
degree 4 same end behav

even

leading coeff 1 open up

$a > 0$ rise left

rise right



Ex. 4) Sketch the graph: $f(x) = -x^3+5x^2+2x-24$

degree 3

odd

opp end behav

lead coeff -1

rise left

fall right

y-int $f(0) = -0^3 + 5(0)^2 + 2(0) - 24$

$= -24$

$0 = -x^3 + 5x^2 + 2x - 24$

$0 = x^3 - 5x^2 - 2x + 24$

factor

poss "a" values

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

remainder thm $f(-2) = (-2)^3 - 5(-2)^2 - 2(-2) + 24$

$= 0$

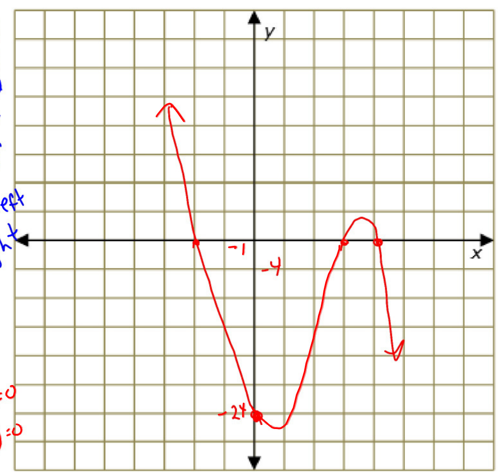
synthetic division

$$\begin{array}{r|rrrr} -2 & 1 & -5 & -2 & 24 \\ & \downarrow & -2 & 14 & -24 \\ \hline & 1 & -7 & 12 & \boxed{0} \end{array}$$

$(x+2)(x^2-7x+12)=0$

$x=-2 \quad (x-4)(x-3)=0$

$x=4 \quad x=3$



When a zero has ***even multiplicity***, the graph touches the x-axis at the related x-intercept, but does not cross it. bounces

When a zero has ***odd multiplicity***, the graph flattens out and crosses the x-axis at the related x-intercept.

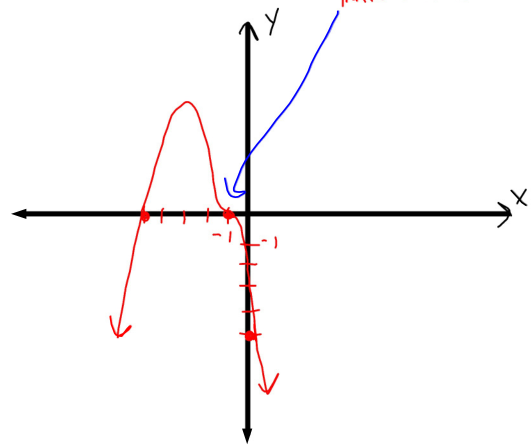
Ex. 5) Sketch the graph: $g(x) = -(x+5)(x+1)^3$ ← mult of 3
flatten and cross

y-int
$g(0) = -(0+5)(0+1)^3$
$= -5$

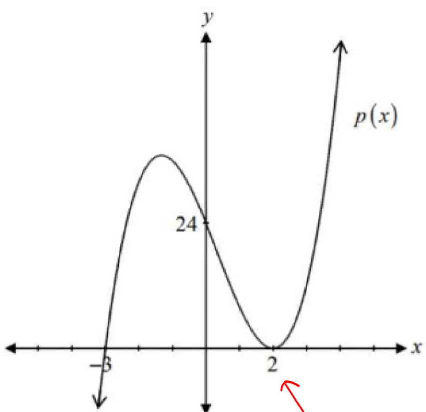
x-int
$x = -5, -1$

degree 4 ← even
same end behav

leading coeff -1 opens down



Ex. 6) Determine the equation of the polynomial function, $p(x)$.



bounce
mult of 2

$p(x) = a(x+3)(x-2)^2$ ← X-ints $-3, 2$
$24 = a(0+3)(0-2)^2$
$24 = 12a$
$2 = a$

y-int
pt
$(0, 24)$
x y
sub in

$\therefore p(x) = 2(x+3)(x-2)^2$

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#3, 4 g, h, i, j
7a, b, 8