

Lesson 1 Sketching Exponential & Logarithmic Functions

Exponential Functions

The function $y = a^x$ is an exponential function.

General Form: $f(x) = a^m$, where $a > 0$ and $a \neq 1$

Recall: Negative Exponent Law For example

$$a^{-n} = \frac{1}{a^n} \quad \text{or} \quad \frac{1}{a^{-n}} = a^n$$

$$2^{-1} = \frac{1}{2}$$

$$\frac{1}{3^{-2}} = 3^2$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \quad \left(\frac{3}{5}\right)^{-3} = \left(\frac{5}{3}\right)^3$$

Logarithmic Functions

The inverse of the exponential function $y = a^x$ is $x = a^y$. This inverse is called a logarithmic function, and is written as $y = \log_a x$ (**Read as:** "y equals the log of x in base a"), where "a" is a positive number other than 1.

recall:
switch
x and
y values

Ex. 1) Sketch $y = 2^x$ and $x = 2^y$ on the same grid.

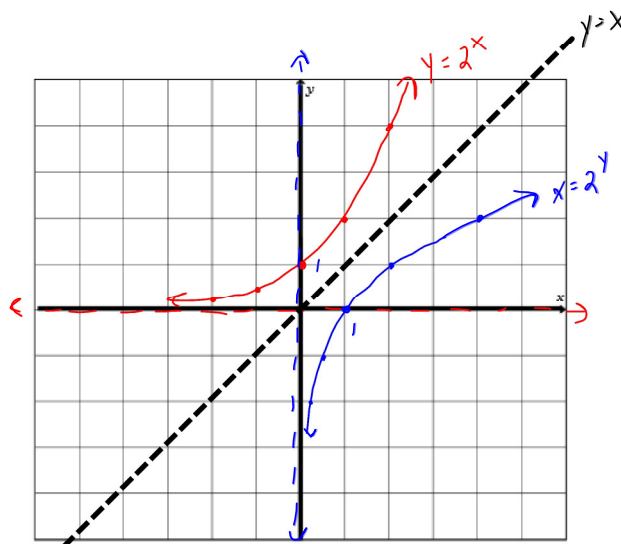
$y = 2^x$

x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

inverse
 $x = 2^y$

x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2

switch x and y values



* reflected over the line $y=x$

Asymptote for $y = 2^x$, $y=0$ becomes $x=0$ for $x = 2^y$

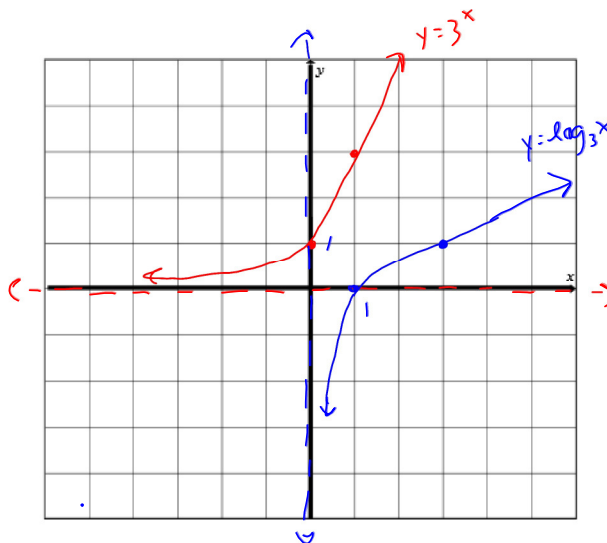
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Ex. 2) Sketch $y = 3^x$ and $y = \log_3 x$ on the same grid.
 ↳ inverse $x = 3^y$

You need a minimum of 2 pts

x	y
0	1
1	3

x	y
1	0
3	1

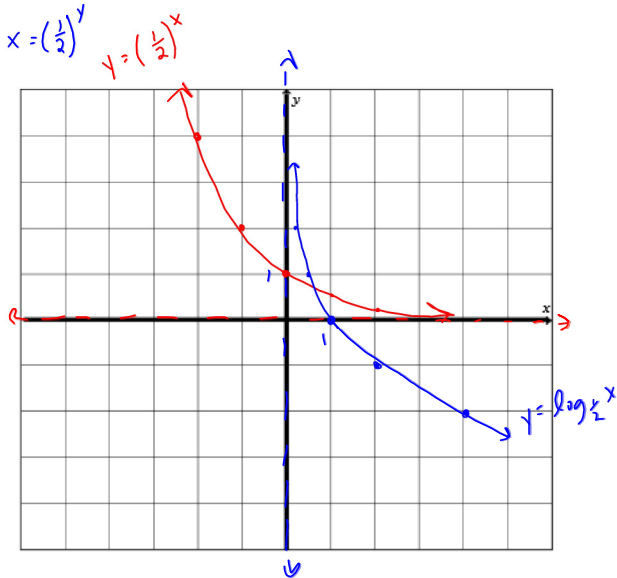


Ex. 3) Sketch $y = (\frac{1}{2})^x$ and $y = \log_{\frac{1}{2}} x$ on the same grid.

↳ inverse same as $x = (\frac{1}{2})^y$

x	y
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$

x	y
4	-2
2	-1
1	0
$\frac{1}{2}$	1
$\frac{1}{4}$	2



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Pre-Calculus 12 Enriched Exponents & Logarithms

Properties of Exponential Functions (base graphs)

a) Zero(s)

none

b) If function is increasing or decreasing
(behaviour from left to right)

$a > 1$, function is increasing

$0 < a < 1$, function decreasing

c) y-intercepts

1
common point
(0, 1)

d) Equations of any asymptotes

$$y = 0$$

e) Domain

$(-\infty, \infty)$

f) Range

$(0, \infty)$

Properties of Logarithmic Functions

Inverse so properties of x become properties of y
and vice versa

a) Zero(s)

1
common point
(1, 0)

b) If function is increasing or decreasing
(behaviour from left to right)

$a > 1$, function increasing

$0 < a < 1$, function decreasing

c) y-intercepts

none

d) Equations of any asymptotes

$$x = 0$$

e) Domain

$(0, \infty)$

" $x > 0$

f) Range

$(-\infty, \infty)$

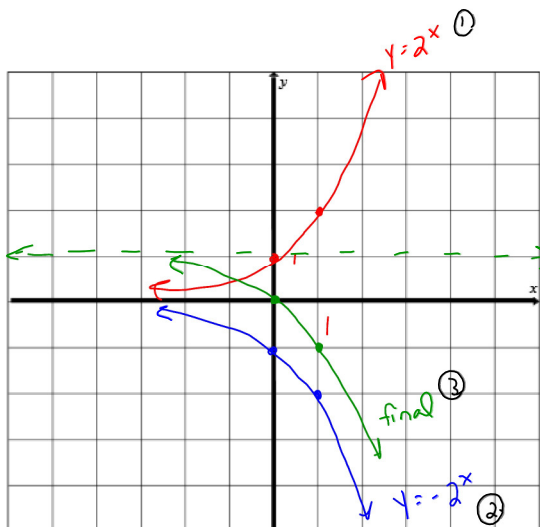
" $y \in \mathbb{R}$

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Sketching, Using Transformations

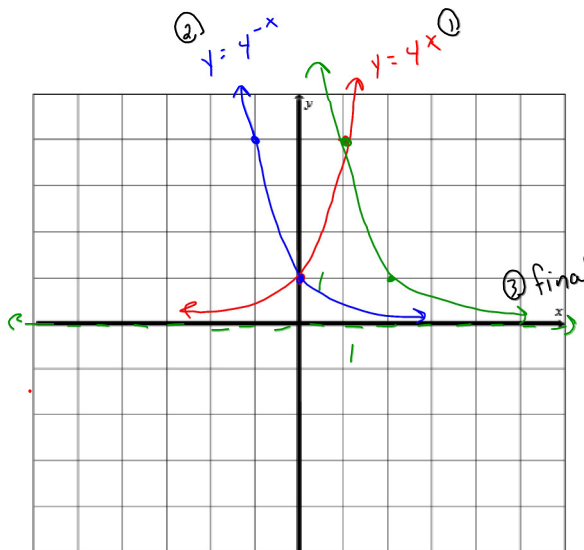
Ex. 4) Sketch $y = -2^x + 1$

- ① Sketch $y = 2^x$
- ② $y = -2^x$
reflection over x-axis
- ③ $y = -2^x + 1$
vertical translation up 1 unit
* asymptote moves with the graph



Ex. 5) Sketch $y = 4^{-(x+2)}$

- $y = 4^x$
- $y = 4^{-x}$
reflection over the y-axis
- $y = 4^{-(x-2)}$
- recall *"b" must be factored out
- \therefore horizontal translation 2 units to the right.



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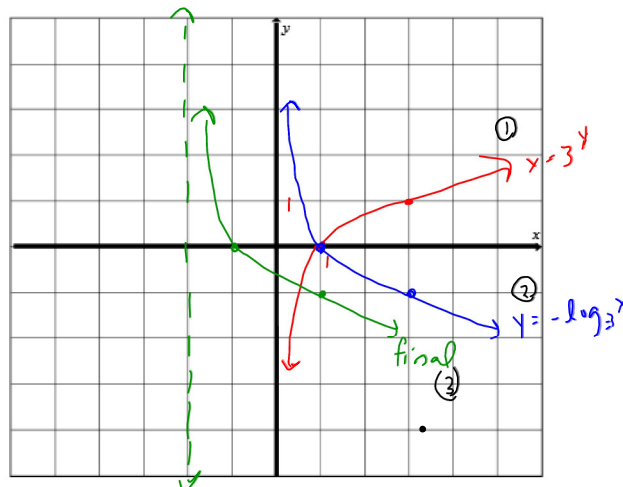
Ex. 6) Sketch $y = -\log_3(x+2)$
 (inverse of $y = 3^x$)

$x = 3^y$

$y = -\log_3 x$
 reflection over the x-axis

$y = -\log_3(x+2)$
 ↑ left 2

* asymptote moves w/ graph

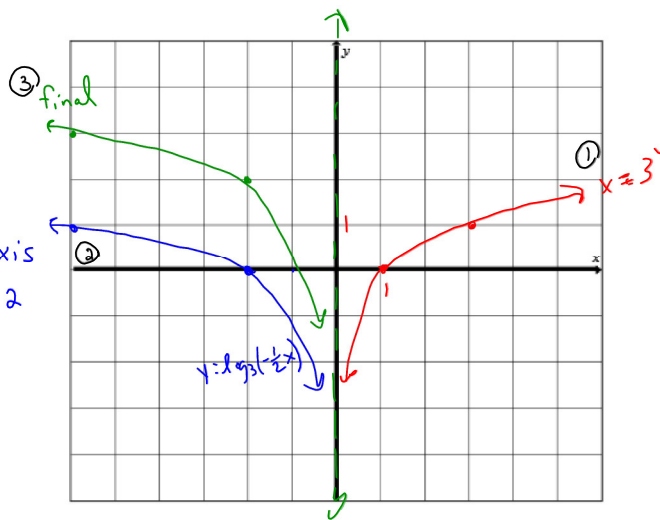


Ex. 7) Sketch $y = \log_3(-\frac{1}{2}x) + 2$

$x = 3^y$

$y = \log_3(-\frac{1}{2}x)$
 reflection over y-axis
 horizontal stretch by 2

$y = \log_3(-\frac{1}{2}x) + 2$
 ↑ w/ 2



pg 160 #4,5
 pg 169 #6
 worksheet #2a,c,e