Pre-Calculus 11 Radicals

Lesson1 Simplifying Radical Expressions

index $\longrightarrow$ $\sqrt[n]{x}$ $\longleftarrow$ radicand

Recall:

Perfect Squares	Square Roots	Cube Roots
$2^2 =$ 4	$\sqrt{4} =$ 2	$\sqrt[3]{8} =$ 2
$3^2 =$ 9	$\sqrt{9} =$ 3	$\sqrt[3]{27} =$ 3
$5^2 =$ 25	$\sqrt{16} =$ 4	$\sqrt[3]{64} =$ 4
$10^2 =$ 100	$\sqrt{100} =$ 10	$\sqrt[3]{1000} =$ 10

Steps to Simplifying Radicals

1. Find ***largest*** perfect square that will divide evenly into the number under your radical sign.
2. Write the number under the radical as a product, including the perfect square as one factor.
3. Separate each number with its own radical sign.
4. Simplify.

Note:

- If you cannot find a perfect square that divides evenly then it is already in simplest form.
- If you do not choose the ***largest*** perfect square, you will need to repeat the process.

Recall: Multiplication Property of Radicals:

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$,

Where n is a natural number, and a and b are real numbers

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Examples – Simplify the following radicals

entire radical

1. $\sqrt{50} = \sqrt{25 \cdot 2}$

complex radical

$\sqrt{25}\sqrt{2}$

$5\sqrt{2}$ ← simple radical

mixed radical

Perfect Squares

4
9
16
25
36

Perfect cubes

8
27
64

2. $\sqrt[3]{54} = \sqrt[3]{27 \cdot 2}$

$\sqrt[3]{27}\sqrt[3]{2}$

$3\sqrt[3]{2}$

Examples – Arrange in order from least to greatest

3. $9\sqrt{2}$, $2\sqrt{6}$, $8\sqrt{3}$

change to entire radicals

$\sqrt{9^2 \cdot 2}$ $\sqrt{2^2 \cdot 6}$ $\sqrt{8^2 \cdot 3}$

$\sqrt{162}$ $\sqrt{24}$ $\sqrt{192}$

least to greatest

$\sqrt{24}$, $\sqrt{162}$, $\sqrt{192}$

or $2\sqrt{6}$, $9\sqrt{2}$, $8\sqrt{3}$

4. $7\sqrt[3]{3}$, $3\sqrt[3]{3}$, $8\sqrt[3]{3}$

like radicals

$3\sqrt[3]{3}$, $7\sqrt[3]{3}$, $8\sqrt[3]{3}$

5. $7\sqrt[3]{2}$, $6\sqrt[4]{5}$, $4\sqrt{5}$

if use estimation, all are between 8 and 9

∴ use calc to put into decimal approximations

8.819, 8.972, 8.944

∴ $7\sqrt[3]{2}$, $4\sqrt{5}$, $6\sqrt[4]{5}$

Similarly to the Multiplication Property of Radicals, we can simplify radicals using the division property of radicals.

Equivalent expressions for any number have the same value.

- $\sqrt{\frac{16}{9}}$ is equivalent to $\frac{\sqrt{16}}{\sqrt{9}}$ because:

$$\sqrt{\frac{16}{9}} = \sqrt{\frac{4}{3} \cdot \frac{4}{3}} = \frac{4}{3} \quad \text{and} \quad \frac{\sqrt{16}}{\sqrt{9}} = \frac{\sqrt{4 \cdot 4}}{\sqrt{3 \cdot 3}} = \frac{4}{3}$$

A similar result is true for any index, n.

Division Property of Radicals:

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, where\ n \in \mathbb{N}\ and\ a, b, \sqrt[n]{a}, \sqrt[n]{b} \in \mathbb{R}, b \neq 0$$

Examples

6. Write $\sqrt[3]{-\frac{40}{81}}$ as a mixed radical.

$$\frac{\sqrt[3]{-40}}{\sqrt[3]{81}} \quad \frac{\sqrt[3]{-8 \cdot 5}}{\sqrt[3]{27 \cdot 3}} \quad \frac{\sqrt[3]{-8} \cdot \sqrt[3]{5}}{\sqrt[3]{27} \cdot \sqrt[3]{3}}$$

$$\frac{-2\sqrt[3]{5}}{3\sqrt[3]{3}} \quad \text{or} \quad -\frac{2}{3}\sqrt[3]{\frac{5}{3}}$$

7. Write $-2\sqrt[3]{\frac{3}{4}}$ as an entire radical

$$\sqrt[3]{(-2)^3\left(\frac{3}{4}\right)}$$

$$\sqrt[3]{-\overset{2}{\cancel{8}}\left(\frac{3}{\cancel{4}}\right)}$$

$$\sqrt[3]{-6}$$

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Examples – Simplifying and defining radicals

8. $\sqrt{5a^2}$ $a \in \mathbb{R}$

$|a|\sqrt{5}$

9. $\sqrt{-27b^9}$

if $b > 0$ ϕ

cannot square root a negative

if $\boxed{b \le 0}$

$\sqrt{-9 \cdot 3 b^4 \cdot b}$

$\sqrt{9}\sqrt{-3}\sqrt{b^4}\sqrt{b}$

$3b^2\sqrt{-3b}$

10. $\sqrt[4]{7z}$ $z \ge 0$

already in simplified form

11. $\sqrt[3]{24x^7}$ $x \in \mathbb{R}$

$\sqrt[3]{8 \cdot 3x^6 x}$ $\sqrt[3]{8}\sqrt[3]{3}\sqrt[3]{x^6}\sqrt[3]{x}$

$2x^2\sqrt[3]{3x}$

Simplifying Radicals
\# 1, 6, 7, 8, 11, 14, 20, 24, 27, 28

Hand #2
due Wed Mar. 20

Assignment: Pg. 100; #3a,c, 4a, c, 5b, 6a, d, 7b, 8a, 10a,b, 11a, 12, 14a,c