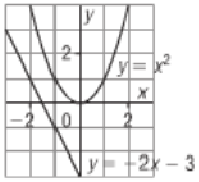
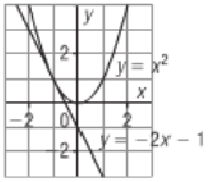
Pre-Calculus 11 Systems & Inequalities

Lesson 5 Solving Systems of Equations Algebraically

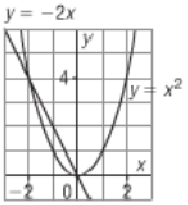
A ***linear-quadratic system*** of equations may have 0, 1 or 2 solutions. The solution is an ordered pair (x, y) that satisfies the two equations in the system (where the graphs intersect).



This system has 0 solutions.



This system has 1 solution.



This system has 2 solutions.

Recall: Solving Linear Systems

Which algebraic strategy would you use to solve each of the following systems? elimination or substitution

System A substitution

$y = 4x - 4$

$2x + 3y = -5$

$2x + 3(4x - 4) = -5$

$2x + 12x - 12 = -5$

$14x = 7$

$x = \frac{1}{2}$

$y = 4(\frac{1}{2}) - 4$

$y = -2$

$\therefore (\frac{1}{2}, -2)$

System B elimination

$3x + 2y = 9$ $\times 2$ $\quad 6x + 4y = 18$

$2x + 3y = 11$ $\times(-3)$ $\quad -6x - 9y = -33$

$-5y = -15$

$y = 3$

add

$2x + 3(3) = 11$

$2x = 2$

$x = 1$

$\therefore (1, 3)$

Steps to solving systems algebraically:

1. Decide which algebraic strategy you want to use to solve the system.
   a. Substitution
   b. Elimination
2. Once you substitute or eliminate one variable, solve for the other.
3. Using the answer found in step 2, substitute that value into one of the original equations and solve for the other variable.
4. Check your answer in both original equations

Example 1
Solve.

$y = \frac{1}{3}x^2 - 3$
$y + x = -3$

$\frac{1}{3}x^2 - 3 + x = -3$

$\frac{1}{3}x^2 + x = 0$ $\times 3$

$\times 3$

$x^2 + 3x = 0$

$x(x+3) = 0$

$x = 0$ $x + 3 = 0$

$x = -3$

2 pts of intersection, determine y for each

$x = 0$

$y + 0 = -3$

$y = -3$

$(0, -3)$

$x = -3$

$y + (-3) = -3$

$y = 0$

$(-3, 0)$

Example 2
Solve.

$y = -2x^2 + 10$
$x - 2y = -15$

$x - 2(-2x^2 + 10) = -15$

$x + 4x^2 - 20 = -15$

$4x^2 + x - 5 = 0$

P -20
S 1
F $-\frac{4}{1}, \frac{5}{1}$

$(4x + 5)(x - 1) = 0$

$4x + 5 = 0$ $x - 1 = 0$

$x = -\frac{5}{4}$ $x = 1$

$x = -\frac{5}{4}$

$y = -2\left(-\frac{5}{4}\right)^2 + 10$

$y = -2\left(\frac{25}{16}\right) + 10$

$y = -\frac{50}{16} + \frac{160}{16}$

$y = \frac{110}{16}$

$y = \frac{55}{8}$

$\left(-\frac{5}{4}, \frac{55}{8}\right)$

$x = 1$

$y = -2(1)^2 + 10$

$y = -2 + 10$

$y = 8$

$(1, 8)$

Example 3

Two unknown numbers are related in the following ways. The square of the first number subtract the second number is equal to 5. The first number is equal to the second number subtract 7. Determine the numbers.

$x^2 - y = 5$

$x = y - 7$

$(y-7)^2 - y = 5$

$(y-7)(y-7) - y = 5$

$y^2 - 7y - 7y + 49 - y = 5$

$y^2 - 15y + 44 = 0$

$(y-11)(y-4) = 0$

$y = 11 \quad y = 4$

$y = 11$

$x = 11 - 7$

$x = 4$

$y = 4$

$x = 4 - 7$

$x = -3$

∴ the numbers are either 4 and 11 or -3 and 4.

Assignment: Pg. 395 #3a, 4, 5a, b, 6