Lesson 1 Quadratic Inequalities in One Variable

Recall:

Inequalities:

$<$ Means less than
$\leq$ Means less than or equal to
$>$ Means greater than
$\geq$ Means greater than or equal to

When the equals sign in a quadratic equation is replaced with an inequality sign, ***a quadratic inequality in one variable*** is formed.

Quadratic Inequalities in One Variable

A quadratic inequality in one variable can be written in general form as:

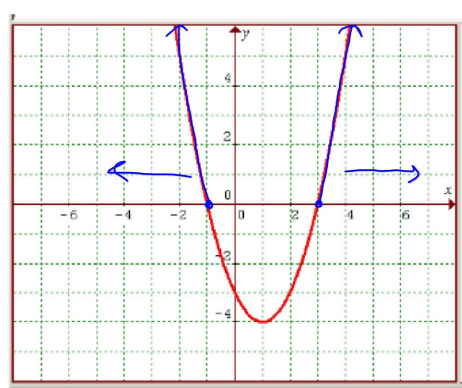
$ax^2 + bx + c < 0$ $\quad$ $ax^2 + bx + c \leq 0$
$ax^2 + bx + c > 0$ $\quad$ $ax^2 + bx + c \geq 0$

Where a, b, and c are constants and $a \neq 0$.

The x-intercepts of the graph of a quadratic function are called the ***critical values*** of the corresponding quadratic inequality.

-1 and 3

Example 1



Solve: $x^2 - 2x - 3 > 0$

$(-\infty, -1) \cup (3, \infty)$ positive y-values

Write the corresponding quadratic equation.

$x^2 - 2x - 3 = 0$

Factor the left hand side and solve each bracket.

$(x-3)(x+1) = 0$

$x = 3 \quad x = -1$

These are the roots (sol'ns) to the quadratic equation, the zeros of the quadratic function, and the critical values of the corresponding quadratic inequality.

positive y - values

The solution of the quadratic inequality $x^2 - 2x - 3 > 0$ are the values of x for which $y > 0$; that is, the values of x for which the graph is above the x-axis.

Using interval notation, the solution is: $(-\infty, -1) \cup (3, \infty)$

Instead of graphing the function each time, we can use a faster way to solve inequalities in one variable.

- Set the corresponding quadratic equation equal to 0 (When multiplying or dividing by a negative value, switch the direction of the inequality symbol!)
- Factor, and set all factors equal to 0 to find the critical values
- Set up a table and test a point in each interval
- State the solution. Use a dotted/dashed line if original question is $<$ *or* $>$. Use a solid line if original question is $\leq$ *or* $\geq$

Use factored form to make a table

$$(x-3)(x+1) > 0, \qquad x = 3 \text{ and } x = -1$$

factors	$x - 3$	−	−	+
	$x + 1$	−	+	+
product	$(x-3)(x+1)$	+	−	+

$-\infty$ -1   3 ∞

critical values (least to greatest)

Test values in each interval and record the sign. Try testing -2, 0, 4

positive

Look at where the intervals satisfy the inequality, where is it greater than 0.

Therefore the solution set is $(-\infty, -1) \cup (3, \infty)$

union (or)

Example 2

Solve: $(2x+1)(x-5) \leq 0$

less than or equal to (negative)

$(2x+1)(x-5)=0$

$2x+1=0 \qquad x-5=0$

$x=-\frac{1}{2} \qquad x=5$

$2x+1$	−	+	+
$x-5$	−	−	+
product $(2x+1)(x-5)$	+	−	+
	$-\infty$ to $-\frac{1}{2}$	$-\frac{1}{2}$ to 5	5 to ∞
test values	-1	0	7

sol'n

$\left[-\frac{1}{2}, 5\right]$

Example 3

Solve: $5x \geq 2(x^2-6)$

$5x = 2(x^2-6)$

$5x = 2x^2-12$

$0 = 2x^2-5x-12$

$0 = (2x+3)(x-4)$

$x=-\frac{3}{2} \qquad x=4$

P -24
S -5
F $-\frac{8}{2}, \frac{3}{1}$

$0 \geq 2x^2-5x-12$

negative

$(2x+3)$	−	+	+
$(x-4)$	−	−	+
product $(2x+3)(x-4)$	+	−	+
	$-\infty$ to $-\frac{3}{2}$	$-\frac{3}{2}$ to 4	4 to ∞

sol'n $\left[-\frac{3}{2}, 4\right]$

For $0 \leq 2x^2-5x-12$

sol'n

$\left(-\infty, -\frac{3}{2}\right] \cup [4, \infty)$

Assignment: Pg. 346; #4a, 5 6a, c 7, 10a