# Sequences and Series: R9, R10

**Arithmetic Sequence:** $t_n = t_1 + d(n-1)$

- $t_1$ **is the first term**
- $d$ *is the common difference*
- $n$ *is the number of terms*

**Arithmetic Series:** $S_n = \frac{n(t_1+t_n)}{2}$ $or$ $\frac{n[2t_1+d(n-1)]}{2}$

- **used to find the sum of all the terms**

**Geometric Sequence:** $t_n = t_1 r^{n-1}$

- $t_1$ **is the first term**
- $r$ **is the common ratio**
- $n$ **is the number of terms**

**Geometric Series:** $s_n = \frac{t_1(1-r^2)}{1-r}$

- **used to find the sum of all the terms**

**Infinite Geometric Series:** $s_\infty = \frac{t_1}{1-r}$

- **If an infinite geometric series converges ($-1 < r < 1$), then you can determine the sum.**

1. Determine the 20th term: $-9, -3, 3, 9, \ldots$

$+6 \quad +6 \quad +6$

$d = t_2 - t_1$
$= -3-(-9)$
$= 6$

$t_{20} = t_1 + d(n-1)$
$= -9 + 6(20-1)$
$= -9 + 6(19)$
$= -9 + 114$
$= 105$

2. Determine the sum of the first 7 terms of an arithmetic sequence where: $t_1 = 5$, $t_7 = 17$

$$S_n = \frac{n(t_1+t_n)}{2}$$
$$S_7 = \frac{7(5+17)}{2}$$
$$S_7 = \frac{7(\cancel{22})^{11}}{\cancel{2}}$$
$$S_7 = 77$$

3. Find the 10th term: $2, -6, 18, \ldots$ ×-3 ×-3 $r=-3$

$$t_n = t_1 r^{n-1}$$
$$t_{10} = 2(-3)^{10-1}$$
$$= 2(-3)^9$$
$$= 2(-19683)$$
$$= -39,366$$

4. Find the sum of the first 12 terms: $3, 12, 48, \ldots$ ×4 ×4 $r=4$

$$S_n = \frac{t_1(1-r^n)}{1-r}$$
$$S_{12} = \frac{3(1-4^{12})}{1-4}$$
$$= \frac{\cancel{3}^{-1}(-16,777,215)}{-\cancel{3}} \Rightarrow S_{12} = +16,777,215$$

5. Find the sum, *if* it exists:

a) $27, -9, 3, -1, \ldots$

$$r = \frac{-9}{27} = \frac{-1}{3}$$

$$S_\infty = \frac{t_1}{1-r} = \frac{27}{1-(-1/3)} = \frac{27}{4/3} = \frac{27}{1} \times \frac{3}{4} = \frac{81}{4} = 20.25$$

b) $4, -8, 16, -32, \ldots$

$$r = \frac{-8}{4} = -2$$

The sum does not exist because r is not between -1 and 1.