The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

Cayley Contest

1. The value of $6 + 4 \div 2$ is

   (A) 5 (B) 6 (C) 7 (D) 8 (E) 9

2. In the diagram, 3 of the 1×1 squares that make up the 4×5 grid are shaded. How many additional 1×1 squares need to be shaded so that one-half of all of the 1×1 squares are shaded?

   (A) 5 (B) 9 (C) 7

   (D) 6 (E) 8

3. The base of a rectangular box measures 2 cm by 5 cm. The volume of the box is 30 cm^3. What is the height of the box?

   (A) 1 cm   (B) 2 cm   (C) 3 cm   (D) 4 cm   (E) 5 cm

4. How many of the four integers 222, 2222, 22 222, and 222 222 are multiples of 3?

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

5. If $\sqrt{100 - x} = 9$, then x equals

   (A) 9 (B) 91 (C) $\sqrt{19}$ (D) 97 (E) 19

6. A large cylinder can hold 50 L of chocolate milk when full. The tick marks show the division of the cylinder into four parts of equal volume. Which of the following is the best estimate for the volume of chocolate milk in the cylinder as shown?

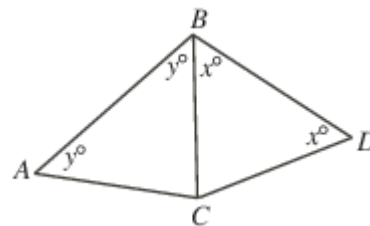   (A) 24 L   (B) 28 L   (C) 30 L

   (D) 36 L   (E) 40 L
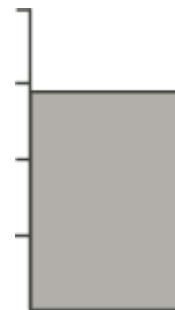
7. The mean (average) of 5 consecutive integers is 9. What is the smallest of these 5 integers?

   (A) 4 (B) 5 (C) 6 (D) 7 (E) 8

8. In the diagram, triangles ABC and CBD are isosceles. The perimeter of $\triangle CBD$ is 19, the perimeter of $\triangle ABC$ is 20, and the length of BD is 7. What is the length of AB?

   (A) 5 (B) 6 (C) 7

   (D) 8 (E) 9

9. If $a = 7$ and $b = 13$, the number of even positive integers less than ab is

   (A) $\dfrac{ab - 1}{2}$ (B) $\dfrac{ab}{2}$ (C) $ab - 1$ (D) $\dfrac{a + b}{4}$

   (E) $(a - 1)(b - 1)$

10. Consider the following flowchart:

$$\boxed{\text{INPUT}} \to \textit{Subtract } 8 \to \boxed{} \to \textit{Divide by } 2 \to \boxed{} \to \textit{Add } 16 \to \boxed{\text{OUTPUT}}$$

If the **OUTPUT** is 32, the **INPUT** must have been
(A) **16**   (B) **28**   (C) **36**   (D) **40**   (E) **32**

11. The operation "∇" is defined by $(a, b)\nabla(c, d) = ac + bd$. For example $(1, 2)\nabla(3, 4) = (1)(3) + (2)(4) = 11$. The value of $(3, 1)\nabla(4, 2)$ is
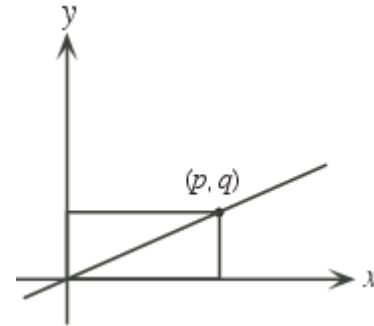(A) **10**   (B) **11**   (C) **13**   (D) **14**   (E) **24**

12. The point (p, q) is on the line $y = \frac{2}{5}x$, as shown.
Also, the area of the rectangle shown is 90. What is the value of p?
(A) **12**   (B) **9**   (C) **10**
(D) **15**   (E) **30**

13. An integer x is chosen so that $3x + 1$ is an even integer. Which of the following must be an odd integer?
(A) $x + 3$ (B) $x - 3$ (C) $2x$ (D) $7x + 4$ (E) $5x + 3$

14. Including the endpoints, how many points on the line segment joining $(-9, -2)$ and $(6, 8)$ have coordinates that are both integers?
(A) 2   (B) 7   (C) 16   (D) 11   (E) 6

15. A bag contains red, blue and purple marbles, and does not contain any other marbles. The ratio of the number of red marbles to the number of blue marbles is $4 : 7$. The ratio of the number of blue marbles to the number of purple marbles is $2 : 3$. There are 32 red marbles in the bag. In total, how many marbles are there in the bag?
(A) 162   (B) 129   (C) 176   (D) 164   (E) 172

16. The *non-negative difference* between two numbers a and b is $a - b$ or $b - a$, whichever is greater than or equal to 0. For example, the non-negative difference between 24 and 64 is 40. In the sequence 88, 24, 64, 40, 24, ..., each number after the second is obtained by finding the non-negative difference between the previous 2 numbers. The sum of the first 100 numbers in this sequence is
(A) **496**   (B) **760**   (C) **752**   (D) **776**   (E) **405**
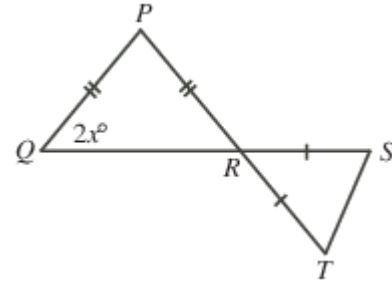
17. A circle with area 36π is cut into quarters and three of the pieces are arranged as shown. What is the perimeter of the resulting figure?
(A) $6\pi + 12$ (B) $9\pi + 12$ (C) $9\pi + 18$
(D) $27\pi + 12$ (E) $27\pi + 24$
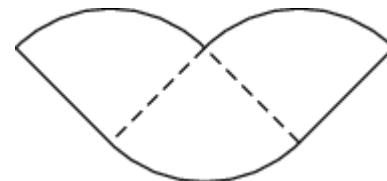
18. In the diagram, R is the point of intersection of PT and QS, $PQ = PR$, and $RS = RT$. If $\angle PQR = 2x°$, then the measure of $\angle RST$, in degrees, is

(A) $45 - x$ (B) $90 + \frac{1}{2}x$ (C) $90 - \frac{1}{2}x$

(D) $45 + 2x$ (E) $90 - x$

19. In a bin at the Cayley Convenience Store, there are 200 candies. Of these candies, 90% are black and the rest are gold. After Yehudi eats some of the black candies, 80% of the remaining candies in the bin are black. How many black candies did Yehudi eat?

(A) 2   (B) 20   (C) 40   (D) 100   (E) 160

20. If $wxyz$ is a four-digit positive integer with $w \neq 0$, the *layer sum* of this integer equals $wxyz + xyz + yz + z$. For example, the layer sum of 4089 is $4089 + 089 + 89 + 9 = 4276$. If the layer sum of $wxyz$ equals 2014, what is the value of $w + x + y + z$?

(A) 12   (B) 15   (C) 11   (D) 13   (E) 10

21. A *lattice point* is a point with integer coordinates. (For example, $(1, 4)$ is a lattice point but $(\frac{3}{2}, 4)$ is not.) The line $y = 3x - 5$ passes through square $PQRS$ as shown in the diagram. If the coordinates of R are $(2009, 2009)$, then the number of lattice points on the line which are inside the square is

(A) 666   (B) 667   (C) 668

(D) 669   (E) 670
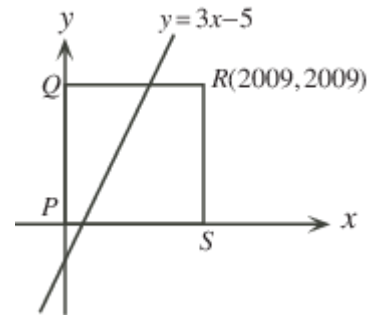
22. In the diagram, AOB is a quarter circle of radius 10 and $PQRO$ is a rectangle of perimeter 26. The perimeter of the shaded region is

(A) $7 + 5\pi$ (B) $13 + 5\pi$ (C) $17 + 5\pi$

(D) $7 + 25\pi$ (E) $17 + 25\pi$
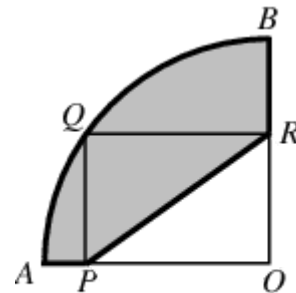
23. In the diagram, points P, Q and R lie on a circle with centre O and radius 12, and point S lies on OR. If $\angle POR = 135°$, the area of trapezoid $OPQS$ is closest to

(A) 216   (B) 144   (C) 108

(D) 112.5   (E) 114.6
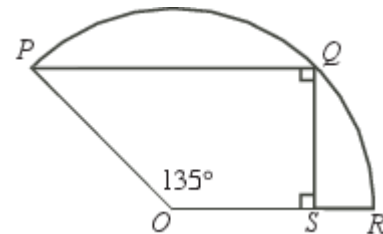
24. A cube has edges of length 1 cm and has a dot marked in the centre of the top face. The cube is sitting on a flat table. The cube is rolled, without lifting or slipping, in one direction so that at least two of its vertices are always touching the table. The cube is rolled until the dot is again on the top face. The length, in centimetres, of the path travelled by the dot is

(A) π (B) 2π (C) $\sqrt{2}\pi$ (D) $\sqrt{5}\pi$

(E) $\left(\dfrac{1+\sqrt{5}}{2}\right)\pi$

25. A semi-circle of radius 8 cm, rocks back and forth along a line. The distance between the line on which the semi-circle sits and the line above is 12 cm. As it rocks without slipping, the semi-circle touches the line above at two points. (When the semi-circle hits the line above, it immediately rocks back in the other direction.) The distance between these two points, in millimetres, is closest to

(A) 55   (B) 53   (C) 51

(D) 49   (E) 47