# The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

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**Fermat Contest**

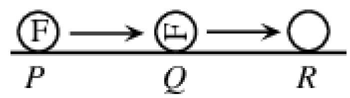
1. The average (mean) of the five numbers 8, 9, 10, 11, 12 is
   (A) 12.5 (B) 8 (C) 9.6 (D) 9 (E) 10

2. If $3 - 5 + 7 = 6 - x$, then $x$ equals
   (A) $-3$ (B) $-1$ (C) 1 (D) 11 (E) 15

3. If $a = \frac{1}{2}$ and $b = \frac{2}{3}$, then $\dfrac{6a + 18b}{12a + 6b}$ equals
   (A) 9 (B) 7 (C) 10 (D) 6 (E) $\frac{3}{2}$

4. How many integers are greater than $\frac{5}{7}$ and less than $\frac{28}{3}$?
   (A) 1 (B) 9 (C) 5 (D) 7 (E) 3

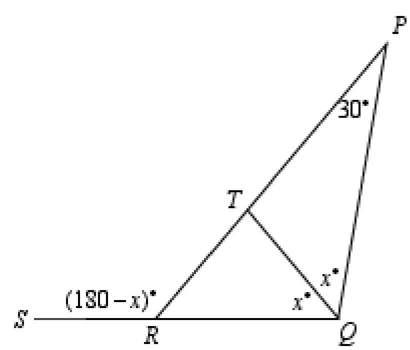
5. A Fermat coin rolls from $P$ to $Q$ to $R$, as shown. If the distance from $P$ to $Q$ is equal to the distance from $Q$ to $R$, what is the orientation of the coin when it reaches $R$?




   (A)  (B)  (C) 
   (D)  (E) 

6. If 50% of $N$ is 16, then 75% of $N$ is
   (A) 12 (B) 6 (C) 20 (D) 24 (E) 40

7. In the diagram, point $T$ is on side $PR$ of $\triangle PQR$ and $QRS$ is a straight line segment. The value of $x$ is
   (A) 55 (B) 70 (C) 75
   (D) 60 (E) 50



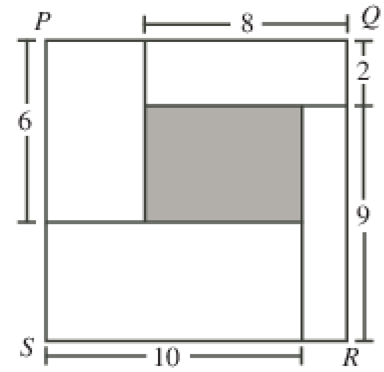

8. In a group of five friends:
   - Amy is taller than Carla.
   - Dan is shorter than Eric but taller than Bob.
   - Eric is shorter than Carla.

   Who is the shortest?
   (A) Amy (B) Bob (C) Carla (D) Dan (E) Eric

9. The regular price for a bicycle is \$320. The bicycle is on sale for 20% off. The regular price for a helmet is \$80. The helmet is on sale for 10% off. If Sandra bought both items on sale, what is her percentage savings on the total purchase?
   (A) 18% (B) 12% (C) 15% (D) 19% (E) 22.5%

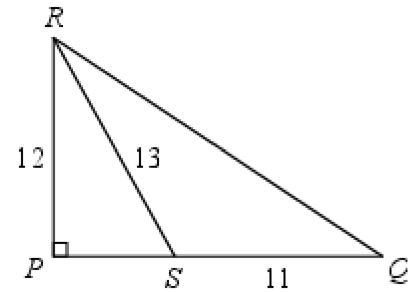
10. In the diagram, $PQRS$ is a square. Square $PQRS$ is divided into five rectangles, as shown. The area of the shaded rectangle is

(A) 49 (B) 28 (C) 22 (D) 57 (E) 16




11. In the diagram, $\triangle PQR$ is right-angled at $P$ and $PR = 12$. If point $S$ is on $PQ$ so that $SQ = 11$ and $SR = 13$, the perimeter of $\triangle QRS$ is

(A) 47 (B) 44 (C) 30 (D) 41 (E) 61




12. If $10^x \cdot 10^5 = 100^4$, what is the value of $x$?

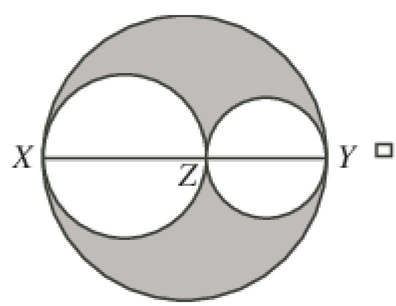
(A) 1 (B) 35 (C) 11 (D) $\frac{4}{5}$ (E) 3

13. In 2004, Gerry downloaded 200 songs. In 2005, Gerry downloaded 360 songs at a cost per song which was 32 cents less than in 2004. Gerry's ***total*** cost each year was the same. The cost of downloading the 360 songs in 2005 was

(A) \$144.00 (B) \$108.00 (C) \$80.00 (D) \$259.20 (E) \$72.00

14. In the diagram, $Z$ lies on $XY$ and the three circles have diameters $XZ$, $ZY$ and $XY$. If $XZ = 12$ and $ZY = 8$, then the ratio of the area of the shaded region to the area of the unshaded region is

(A) 12 : 25 (B) 12 : 13 (C) 1 : 1 (D) 1 : 2 (E) 2 : 3




15. The cookies in a cookie jar contain a total of 100 raisins. All but one of the cookies are the same size and contain the same number of raisins. One cookie is larger and contains one more raisin than each of the others. The number of cookies in the jar is between 5 and 10, inclusive. How many raisins are in the larger cookie?

(A) 10 (B) 11 (C) 20 (D) 17 (E) 12

16. Positive integers $a$ and $b$ satisfy $ab = 2010$. If $a > b$, the smallest possible value of $a - b$ is

(A) 37 (B) 119 (C) 191 (D) 1 (E) 397

17. If $a$ and $b$ are two distinct numbers with $\dfrac{a+b}{a-b} = 3$, then $\dfrac{a}{b}$ equals

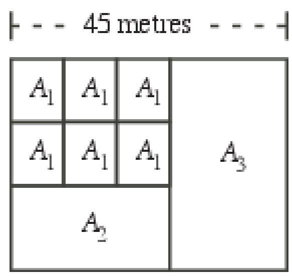
(A) $-1$ (B) 3 (C) 1 (D) 2 (E) 5

18. There are two values of $k$ for which the equation $x^2 + 2kx + 7k - 10 = 0$ has two equal real roots (that is, has exactly one solution for $x$). The sum of these values of $k$ is
(A) $0$ (B) $-3$ (C) $3$ (D) $-7$ (E) $7$

19. Max and Minnie each add up sets of three-digit positive integers. Each of them adds three different three-digit integers whose nine digits are all different. Max creates the largest possible sum. Minnie creates the smallest possible sum. The difference between Max's sum and Minnie's sum is
(A) $594$ (B) $1782$ (C) $1845$ (D) $1521$ (E) $2592$

20. Katie and Sarah run at different but constant speeds. They ran two races on a track that measured 100 m from start to finish. In the first race, when Katie crossed the finish line, Sarah was 5 m behind. In the second race, Katie started 5 m behind the original start line and they ran at the same speeds as in the first race. What was the outcome of the second race?
(A) Katie and Sarah crossed the finish line at the same time.
(B) When Katie crossed the finish line, Sarah was 0.25 m behind.
(C) When Katie crossed the finish line, Sarah was 0.26 m behind.
(D) When Sarah crossed the finish line, Katie was 0.25 m behind.
(E) When Sarah crossed the finish line, Katie was 0.26 m behind.

21. A farmer has a rectangular field with width 45 metres. He divides the field into smaller rectangular animal enclosures in three different sizes, as shown.




Each enclosure labelled $A_1$ has the same dimensions. Also, the area of the enclosure labelled $A_2$ is 4 times the area of $A_1$, and the area of the enclosure labelled $A_3$ is 5 times the area of $A_1$. The lines in the diagram represent fences. The total length of all the fences is 360 metres. The area of $A_1$, in square metres, is closest to
(A) $143.4$ (B) $150$ (C) $175.2$ (D) $162.7$ (E) $405$

22. Megan and Shana race against each other with the winner of each race receiving $x$ gold coins and the loser receiving $y$ gold coins. (There are no ties and $x$ and $y$ are integers with $x > y > 0$.) After several races, Megan has 42 coins and Shana has 35 coins. Shana has won exactly 2 races. The value of $x$ is
(A) $3$ (B) $7$ (C) $5$ (D) $6$ (E) $4$

23. Let $t_n$ equal the integer closest to $\sqrt{n}$.For example, $t_1 = t_2 = 1$ since $\sqrt{1} = 1$ and $\sqrt{2} \approx 1.41$ and $t_3 = 2$ since $\sqrt{3} \approx 1.73$. The sum
$\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \frac{1}{t_4} + \cdots + \frac{1}{t_{2008}} + \frac{1}{t_{2009}} + \frac{1}{t_{2010}}$ equals
(A) $88\frac{1}{6}$ (B) $88\frac{1}{2}$ (C) $88\frac{2}{3}$ (D) $88\frac{1}{3}$ (E) $90$

24. For any real number $x$, $\lfloor x \rfloor$ denotes the largest integer less than or equal to $x$. For example, $\lfloor 4.2 \rfloor = 4$ and $\lfloor 0.9 \rfloor = 0$.
If $S$ is the sum of all integers $k$ with $1 \leq k \leq 999{,}999$ and for which $k$ is divisible by $\lfloor \sqrt{k} \rfloor$, then $S$ equals

(A) 999 500 000 (B) 999 000 000 (C) 999 999 000
(D) 998 999 500 (E) 998 500 500

25. For each positive digit $D$ and positive integer $k$, we use the symbol $D_{(k)}$ to represent the positive integer having exactly $k$ digits, each of which is equal to $D$. For example, $2_{(1)} = 2$ and $3_{(4)} = 3333$. There are $N$ quadruples $(P, Q, R, k)$ with $P$, $Q$ and $R$ positive digits, $k$ a positive integer with $k \leq 2018$, and
$P_{(2k)} - Q_{(k)} = \left(R_{(k)}\right)^2$. The sum of the digits of $N$ is

(A) 10 (B) 9 (C) 11 (D) 12 (E) 13