Lesson 2
Rational Functions with Points of Discontinuity

The graph of a rational function can be discontinuous at a value of x without having an asymptote. This will occur if the numerator and denominator have a common factor.

Steps:

1. Factor the numerator and denominator.
2. Determine vertical and horizontal asymptotes and/or holes.
3. Plot any x-intercepts.
4. Plot the y-intercept.
5. Use smooth curves to complete the graph, approaching the asymptotes as x approaches $\pm\infty$.

Ex. 1) Sketch: $f(x) = \frac{x^2-4}{x+2}$.

State the domain and range.

$f(x) = \frac{(x-2)\cancel{(x+2)}}{\cancel{x+2}}$

$f(x) = x-2$

common factor

$x+2$

$\therefore$ point of discontinuity (hole)

when $x+2=0$

$x=-2$

sub $x=-2$ into

$f(x) = x-2$

$f(-2) = -2-2$

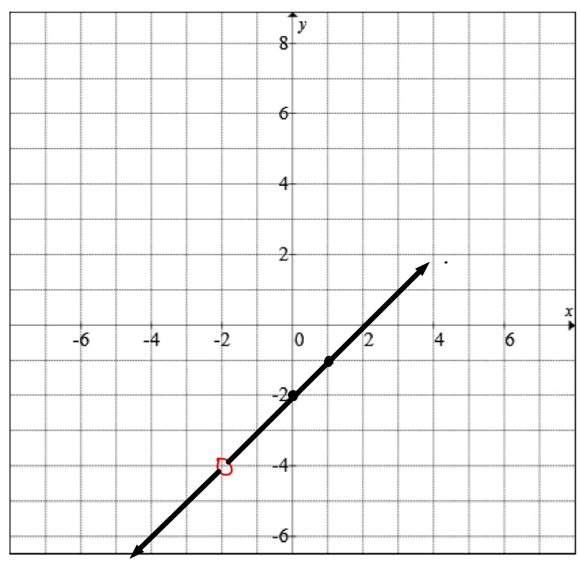
$= -4$

p.o.d.

$(-2,-4)$

Sketch $f(x) = x-2$

line w/ slope of 1, y-int @ -2



D: $x \neq -2$

R: $y \neq -4$

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Ex. 2) Sketch $f(x) = \frac{x-3}{x^2-2x-3}$.
State the domain and range.

$f(x) = \frac{\cancel{x-3}}{(\cancel{x-3})(x+1)}$

$\boxed{f(x) = \frac{1}{x+1}}$

y-int

$f(0) = \frac{1}{0+1}$

$= 1$

x-int

$1 \neq 0$

no x-int

V.A.

$x = -1$

H.A.

$y = 0$ $m < n$

p.o.d.

$x = 3$

$f(3) = \frac{1}{3+1}$

$= \frac{1}{4}$

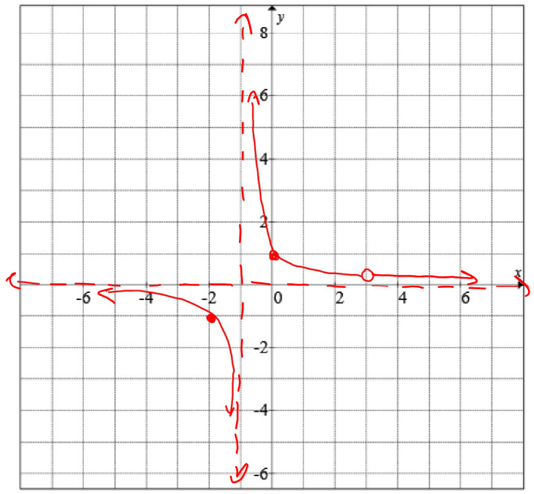
$(3, \frac{1}{4})$

$f(-2) = \frac{1}{-2+1}$

$= -1$

D: $x \neq -1, 3$

R: $y \neq 0, \frac{1}{4}$



Ex. 3) Sketch the graph of $f(x) = \frac{x^2-1}{x^2+x-2}$.
State the domain and range.

$f(x) = \frac{(\cancel{x-1})(x+1)}{(x+2)(\cancel{x-1})}$

$\boxed{f(x) = \frac{x+1}{x+2}}$

y-int

$f(0) = \frac{0+1}{0+2}$

$= \frac{1}{2}$

x-int

$x+1 = 0$

$x = -1$

V.A.

$x + 2 = 0$

$x = -2$

p.o.d. when

$x = 1$

$f(1) = \frac{1+1}{1+2}$

$= \frac{2}{3}$

$(1, \frac{2}{3})$

H.A. $y = 1$

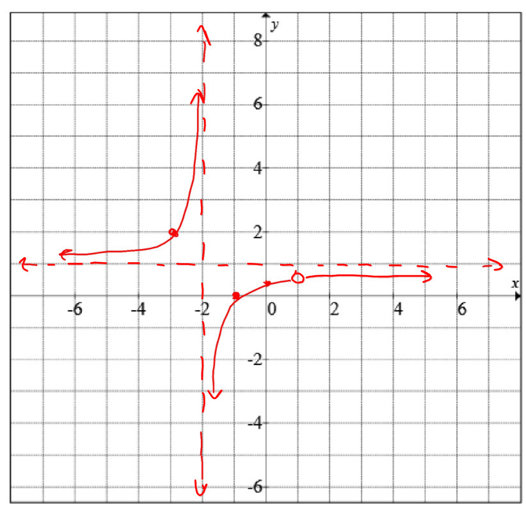
$m = n$

$y = \frac{a}{b}$

$y = \frac{1}{1}$

$f(-3) = \frac{-3+1}{-3+2}$

$= 2$



D: $x \neq -2, 1$

R: $y \neq \frac{2}{3}, 1$

① worksheet

② pg. 142 # 2, 5

③ previous exam questions

④ matching graphs