

L5 Rational Exponents

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Lesson 5 Rational Exponents

Complete each table below. Use a calculator to complete the second column.

\wedge
 y^x
 x^y
 $1 \wedge (1 \div 2)$
 $1 y^x (1 \div 2)$

x	$x^{\frac{1}{2}} = \sqrt{x}$
1	$1^{\frac{1}{2}} = 1$
4	$4^{\frac{1}{2}} = 2$
9	3
16	4
25	5

x	$x^{\frac{1}{3}} = \sqrt[3]{x}$
1	$1^{\frac{1}{3}} = 1$
8	$8^{\frac{1}{3}} = 2$
27	3
64	4
125	5

Rational Number – A number that can be written in the form $\frac{m}{n}$ where m and n are integers ($n \neq 0$).

To evaluate rational exponents we change from exponential form to radical form.

Square Root: $4^{\frac{1}{2}} = \sqrt{4} = 2$

Cube Root: $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

Rational Exponents: $a^{\frac{1}{n}} = \sqrt[n]{a}$

$a^{\frac{1}{n}}$ exists if n is an odd integer: $\sqrt[3]{8} = 2$, $\sqrt[3]{-8} = -2$

$a^{\frac{1}{n}}$ exists if n is even, only if a is positive: $\sqrt{4} = 2$, $\sqrt{-4} = \emptyset$

Examples: Evaluating Powers of the Form $a^{\frac{1}{n}}$

Evaluate each power without using a calculator

1. $27^{\frac{1}{3}}$ $\sqrt[3]{27} = 3$

2. $(-64)^{\frac{1}{3}}$
 $\sqrt[3]{-64}$
 -4

3. $\left(\frac{4}{9}\right)^{\frac{1}{2}}$
 $\sqrt{\frac{4}{9}}$
 $\frac{\sqrt{4}}{\sqrt{9}}$
 $\frac{2}{3}$

Try $25^{\frac{1}{2}} = 5$
 $(-8)^{\frac{1}{3}} = -2$

Extend $(a^m)^n$ to rational exponents.

$$4^{\frac{3}{2}} = (4^{\frac{1}{2}})^3 = (\sqrt{4})^3 = 2^3 = 8$$

$$a^{\frac{m}{n}} = (a^{\frac{1}{n}})^m = (\sqrt[n]{a})^m \text{ where } a \geq 0 \text{ if } n \text{ is even}$$

Note: $\frac{m}{n}$ Must be in reduced form

Examples: Evaluating Powers of the Form $a^{\frac{m}{n}}$

4. Convert $\sqrt[4]{x^3}$ to exponential form.

$$\times \frac{3}{4}$$

Try

$$5\sqrt{x^2}$$

$$3\sqrt[2]{7}$$

$$\times \frac{2}{5}$$

$$2\frac{2}{3}$$

5. Convert $7^{\frac{2}{3}}$ to radical form.

$$3\sqrt[3]{7^2} \text{ or } (3\sqrt[3]{7})^2$$

Try

$$\frac{2}{5}$$

$$\times \frac{3}{4}$$

$$3$$

6. Determine exact value without using the calculator:

a) $32^{\frac{2}{5}}$

$$(\sqrt[5]{32})^2$$

$$2^2$$

$$4$$

b) $0.04^{\frac{3}{2}}$

$$\left(\frac{4}{100}\right)^{\frac{3}{2}}$$

$$\left(\frac{1}{25}\right)^{\frac{3}{2}}$$

$$\left(\sqrt{\frac{1}{25}}\right)^3$$

$$\left(\frac{1}{5}\right)^3$$

$$\frac{1}{125}$$

reduce

c) $(-32)^{0.4}$

$$(-32)^{\frac{4}{10}}$$

$$(-32)^{\frac{2}{5}}$$

$$(\sqrt[5]{-32})^2$$

$$(-2)^2$$

$$4$$

d) $-27^{\frac{1}{3}}$

$$-\sqrt[3]{27}$$

$$-3$$

different than

$$(-27)^{\frac{1}{3}}$$

$$\sqrt[3]{-27}$$

$$-3$$

e) $32^{\frac{3}{5}}$

$$(\sqrt[5]{32})^3$$

$$2^3$$

$$8$$

f) $125^{\frac{2}{3}}$

$$(\sqrt[3]{125})^2$$

$$5^2$$

$$25$$

Try $64^{\frac{2}{3}}$ $\sqrt[3]{64}^2$ 4^2 16

$$(-36)^{\frac{1}{2}}$$

$$\sqrt{-36}$$

$$\emptyset$$

$$-36^{\frac{1}{2}}$$

$$-\sqrt{36}$$

$$-6$$

$$16^{\frac{3}{4}}$$

$$4\sqrt[4]{16}^3$$

$$2^3$$

$$8$$