

## Lesson 3 Mixed and Entire Radicals

Radicals can be written in different forms in order to simplify complex radicals.

$\sqrt{25 \cdot 4}$  is equivalent to  $\sqrt{25} \cdot \sqrt{4}$  because:

$$\begin{array}{c} \sqrt{100} \\ 10 \end{array}$$

$$\begin{array}{c} 5 \cdot 2 \\ 10 \end{array}$$

Similarly,  $\sqrt[3]{8 \cdot 27}$  is equivalent to  $\sqrt[3]{8} \cdot \sqrt[3]{27}$  because:

$$\begin{array}{c} \sqrt[3]{216} \\ 6 \end{array}$$

$$\begin{array}{c} 2 \cdot 3 \\ 6 \end{array}$$

**Multiplication Property of Radicals:**

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b},$$

Where  $n$  is a natural number, and  $a$  and  $b$  are real numbers

**Steps to Simplifying Radicals**

**Example 1**

Simplify  $\sqrt{18}$  ← entire radical

*change to a mixed radical*

$2^2 = 4$   
 $3^2 = 9$   
 $4^2 = 16$

1. Find the **largest** perfect square that will divide evenly into the number under your radical sign.

9

2. Write the number under the radical as a product of the perfect square and its corresponding number.

$\sqrt{9 \cdot 2}$

3. Separate each number with its own radical sign.

$\sqrt{9} \cdot \sqrt{2}$   
 square root

4. Simplify.

$3\sqrt{2}$  ← mixed radical

**Note:**

- If you cannot find a perfect square that divides evenly then it is already in simplest form. *(simple radical)*
- If you do not choose the **largest** perfect square, you will need to repeat the process.

72 has 9 and 36 as perfect square factor. we choose 36

$\sqrt{72}$   
 $\sqrt{36 \cdot 2}$   
 $\sqrt{36} \cdot \sqrt{2}$   
 $6\sqrt{2}$

*repeat*  $\sqrt{72}$   
 $\sqrt{9} \sqrt{8}$   
 $3\sqrt{8}$  ← has a perfect square factor of 4  
 $3\sqrt{4 \cdot 2}$   
 $3\sqrt{4} \sqrt{2}$   
 $3(2)\sqrt{2}$   
 $6\sqrt{2}$

**Example 2: Writing Radicals in Simplest Form**

Simplify each radical.

a)  $\sqrt{63}$   
 $\sqrt{9 \cdot 7}$   
 $\sqrt{9} \cdot \sqrt{7}$   
 $3\sqrt{7}$

b)  $\sqrt{75}$   
 $\sqrt{25 \cdot 3}$   
 $\sqrt{25} \cdot \sqrt{3}$   
 $5\sqrt{3}$

$2^2 = 4$   
 $3^2 = 9$   
 $4^2 = 16$   
 $5^2 = 25$   
 $6^2 = 36$

c)  $\sqrt[3]{108}$  ← entire  
 $\sqrt[3]{27 \cdot 4}$   
 $\sqrt[3]{27} \cdot \sqrt[3]{4}$   
 $3\sqrt[3]{4}$  ← mixed

d)  $\sqrt[3]{56}$   
 $\sqrt[3]{8 \cdot 7}$   
 $2\sqrt[3]{7}$

$2^3 = 8$   
 $3^3 = 27$   
 $4^3 = 64$

**Example 3 - Writing Mixed Radicals as Entire Radicals**

Write each mixed radical as an entire radical.

a)  $7\sqrt{3}$  ← mixed  
 $\sqrt{7^2 \cdot 3}$   
 $\sqrt{49 \cdot 3}$   
 $\sqrt{147}$

b)  $2\sqrt[3]{4}$   
 $\sqrt[3]{2^3 \cdot 3\sqrt[3]{4}}$   
 $\sqrt[3]{8 \cdot 3\sqrt[3]{4}}$   
 $\sqrt[3]{32}$

c)  $2\sqrt{7}$  ←  $\sqrt{2^2 \cdot 7}$   
 $\sqrt{4 \cdot 7}$   
 $\sqrt{28}$