

Lesson 2 Mixed and Entire Radicals

Multiplication Property of Radicals:

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b},$$

$$\text{ex. } \sqrt{10} = \sqrt{2} \cdot \sqrt{5}$$

Where n is a natural number, and a and b are real numbers

Mixed radicals are those where the entire number is not under the radical symbol.

$$\text{ex. } 3\sqrt{11}, 7^3\sqrt{35}, 9^4\sqrt{211}$$

Entire radicals are those where the entire number is under the radical symbol.

$$\text{ex. } \sqrt{17}, \sqrt[3]{32}, \sqrt[4]{162}$$

Simple radicals are those where the radicand doesn't have any square roots, cube roots, 4th roots, etc (dependent on the index) as factors.

Complex radicals are those where the radicand has a square root, cube root, 4th roots, etc factor (dependent on the index).

Radicals can be written in different forms in order to simplify complex radicals.

$$\begin{array}{ccc} \sqrt{25 \cdot 4} \text{ is equivalent to } & \sqrt{25} \cdot \sqrt{4} & \\ \sqrt{100} & 5 \cdot 2 & \\ 10 & = & 10 \end{array}$$

$$\begin{array}{ccc} \text{Similarly, } \sqrt[3]{8 \cdot 27} \text{ is equivalent to } & \sqrt[3]{8} \cdot \sqrt[3]{27} & \\ \sqrt[3]{216} & 2 \cdot 3 & \\ 6 & = & 6 \end{array}$$

Steps to Simplifying Radicals

Example 1

Simplify $\sqrt{18}$ ← entire radical
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 complex radical

- $2^2 = 4$
- $3^2 = 9$
- $4^2 = 16$
- $5^2 = 25$
- $6^2 = 36$

- Determine the **largest** perfect square that will divide evenly into the radicand. largest

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- Write the number under the radical as a product of the perfect square and its corresponding number.

$$\sqrt{9 \cdot 2}$$

- Separate each number with its own radical sign.

$$(\sqrt{9}) \sqrt{2}$$

- Simplify.

$$\underbrace{3\sqrt{2}}_{\text{mixed radical}} \leftarrow \text{simple radical}$$

Note:

- If the radicand does not have a perfect square factor then it is already in simplest form.
- If you do not choose the **largest** perfect square, you will need to repeat the process.

Example 2: Writing Radicals in Simplest Form

Simplify each radical.

Perfect cubes
 $2^3 = 8$
 $3^3 = 27$
 $4^3 = 64$

a) $\sqrt{63}$
 $\sqrt{9 \cdot 7}$
 $\sqrt{9} \cdot \sqrt{7}$
 $3\sqrt{7}$

c) $\sqrt[3]{108}$
 $\sqrt[3]{27 \cdot 4}$
 $\sqrt[3]{27} \cdot \sqrt[3]{4}$
 $3\sqrt[3]{4}$

b) $\sqrt{75}$
 $\sqrt{25 \cdot 3}$
 $\sqrt{25} \cdot \sqrt{3}$
 $5\sqrt{3}$

d) $\sqrt[3]{56}$ ← exact value
 $\sqrt[3]{8 \cdot 7}$
 $\sqrt[3]{8} \cdot \sqrt[3]{7}$
 $2\sqrt[3]{7}$

$\sqrt{75} \approx 8.66$
 $5\sqrt{3} \approx 8.66$ approx value
 the value doesn't change

Example 3 - Writing Mixed Radicals as Entire Radicals

Write each mixed radical as an entire radical.

a) $7\sqrt{3}$
 $\sqrt{7^2 \cdot 3}$
 $\sqrt{49 \cdot 3}$
 $\sqrt{147}$

c) $2\sqrt[3]{4}$
 $\sqrt[3]{2^3 \cdot 4}$
 $\sqrt[3]{8 \cdot 4}$
 $\sqrt[3]{32}$

b) $2\sqrt{7}$
 $\sqrt{2^2 \cdot 7}$
 $\sqrt{4 \cdot 7}$
 $\sqrt{28}$

pg 89 # 1c, e, g, i
 2 e, g
 3 c, g, i
 4 b, e, f, g
 5 b, d, h, j
 6 c, e, i
 9 b, c, h
 10 b, d, i