Lesson 4 Restrictions on Composite Functions

Examples

1. Given the functions $f(x) = x + 1$ and $g(x) = 4 - x^2$,
   a) state the domain and range of each function.

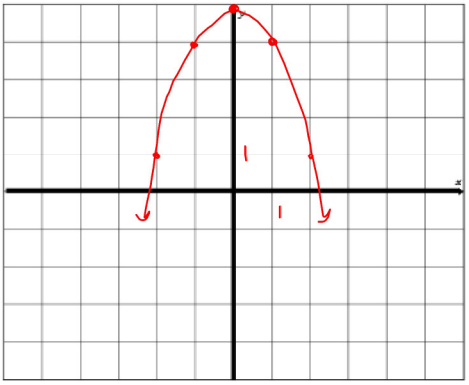
	Domain	Range
$f(x)$	$x \in \mathbb{R}$	$y \in \mathbb{R}$
$g(x)$	$x \in \mathbb{R}$	$y \leq 4$

b) sketch a graph of $y = f(g(x))$. State the domain and range of the new function.

$f(x) = x + 1$

$f(g(x)) = 4 - x^2 + 1$

$= -x^2 + 5$



D: $(-\infty, \infty)$

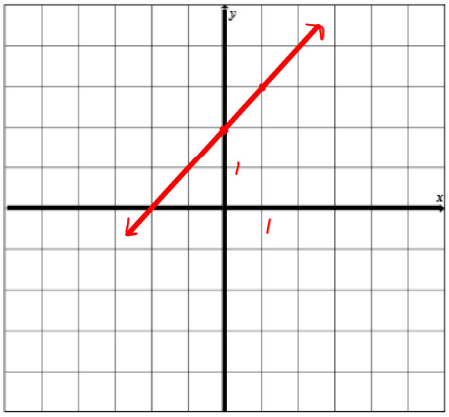
R: $(-\infty, 5]$

c) sketch a graph of $y = f(f(x))$. State the domain and range of the new function.

$f(x) = x + 1$

$f(f(x)) = x + 1 + 1$

$= x + 2$



D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

Pre-Calculus 12 Enriched Combining Functions

The domain of the composite function, $f(g(x))$, is the set of all values of x in the domain of g such that $g(x)$ is in the domain of f.

Steps for finding the domain of composite functions without sketching a graph

1. Determine the domain of the inside functions. If there are any restrictions, keep them.
2. Determine the domain of the composite function.
   - If there are restrictions, add them to the restrictions from step 1.
   - If there is an overlap, use the more restrictive domain.

Examples

2. Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 4$, determine an explicit equation for each of the following and then state the domain of the new function.

a) $g(f(x))$

$g(x) = x^2 - 4$

$g(\sqrt{x}) = (\sqrt{x})^2 - 4$

$= x - 4$

Domain of $f(x)$ is $x \geq 0$

$\therefore$ domain of $g(f(x))$ is $x \geq 0$

b) $f(g(x))$

$f(x) = \sqrt{x}$

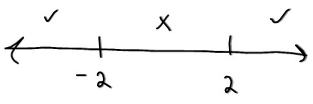
$f(x^2 - 4) = \sqrt{x^2 - 4}$

Domain of $g(x)$ is $x \in \mathbb{R}$

Domain of $f(g(x))$ is

$x^2 - 4 \geq 0$

$x^2 \geq 4$



$\therefore$ D: $(-\infty, -2] \cup [2, \infty)$

Pre-Calculus 12 Enriched Combining Functions

3. Determine possible functions f and g so that $y = f(g(x))$ (inside)

a) $y = (x-2)^2$ (inside)

$g(x) = x - 2$

$f(x) = x^2$

b) $y = \sqrt{x+3}$

$g(x) = x + 3$

$f(x) = \sqrt{x}$

4. Given $f(x) = \frac{1}{x+3}$ and $g(x) = x^2 - 4x$, determine an explicit equation for each composite function below and state the domain of the new function.

a) $g(f(x))$

$g\left(\frac{1}{x+3}\right) = \left(\frac{1}{x+3}\right)^2 - 4\left(\frac{1}{x+3}\right)$

D: $x \neq -3$

b) $f(g(x))$

$f(x^2-4x) = \frac{1}{x^2-4x+3}$

$= \frac{1}{(x-3)(x-1)}$

D: $x \neq 1, 3$

Domain of $g(x)$ is $(-\infty, \infty)$

but $g(x) \neq -3$

$\therefore x^2 - 4x \neq -3$

solving $x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3 \quad x = 1$