Lesson Remainder Theorem

Remainder Theorem:

If $P(x)$ is a polynomial, then $P(a)$ is equal to the remainder when $P(x)$ is divided by $x-a$.

Note:

$x-a$ is a factor of $P(x)$ if the remainder is 0

Ex. 1) Evaluate $f(x) = x^4 - 10x^2 - 2x + 4$ at $f(-3)$.

$f(-3) = (-3)^4 - 10(-3)^2 - 2(-3) + 4$

$= 1$ ← remainder

∴ By the remainder thm, the remainder is 1 when $f(x)$ is divided by $x+3$

compare
By division

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & -10 & -2 & 4 \\ & \downarrow & -3 & 9 & 3 & -3 \\ \hline & 1 & -3 & -1 & 1 & \boxed{1} \end{array}$$

← remainder

Ex. 2) Find the remainder, using only the remainder theorem, when $x^4 - 2x^3 + 5x + 2$ is divided by $x + 1$.

Let $P(x) = x^4 - 2x^3 + 5x + 2$

then $P(-1) = (-1)^4 - 2(-1)^3 + 5(-1) + 2$

$= 0$

$\therefore$ the remainder is 0

which means $x+1$ is a factor of $x^4 - 2x^3 + 5x + 2$

Ex. 3) Determine the value of k when $2x^2 + kx - 5$ is divided by $x - 3$ if the remainder is 7.

Let $P(x) = 2x^2 + kx - 5$

a

$P(3) = 7$

sub in

$P(3) = 2(3)^2 + k(3) - 5$

$7 = 18 + 3k - 5$

$-6 = 3k$

$-2 = k$

Ex. 4) When $P(x) = kx^3 + mx^2 + x - 2$ is divided by $x - 1$, the remainder is 6. When $P(x)$ is divided by $x + 2$, the remainder is 12. Determine the values of k and m.

create 2 eqns

① $P(1) = k(1)^3 + m(1)^2 + 1 - 2$

$6 = k + m - 1$

$7 = k + m$ ①

② $P(-2) = k(-2)^3 + m(-2)^2 - 2 - 2$

$12 = -8k + 4m - 4$

$16 = -8k + 4m$

$\div 4$ $\quad 4 = -2k + m$ ②

system of eqns

① − ②

$7 = k + \cancel{m}$

$-(4 = -2k + \cancel{m})$

elimination method

$3 = 3k$

$k = 1$

sub $k=1$ into ①

$7 = 1 + m$

$6 = m$

pg. 124
\# 6a, b, d, f
7a, b, d
8a, d
10
11