

PROBLEM SET 3.9

1. Each edge of a variable cube is increasing at the rate of 3 inches per second. How fast is the volume of the cube increasing when an edge is 10 inches long?

2. Assuming that a soap bubble retains its spherical shape as it expands, how fast is its radius increasing when its radius is 2 inches, if air is blown into it at the rate of 4 cubic inches a second?

3. An airplane, flying horizontally at an altitude of 1 mile, passes directly over an observer. If the constant speed of the plane is 240 miles per hour, how fast is its distance from the observer increasing 30 seconds later? *Hint:* Use Figure 8 and note that in 30 seconds ($\frac{1}{120}$ hour), the plane goes 2 miles.

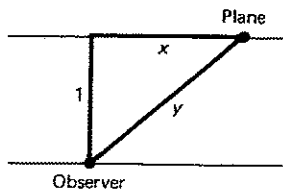


FIGURE 8

4. A student is using a straw to drink from a conical paper cup, whose axis is vertical, at the rate of 3 cubic centimeters a second. If the height of the cup is 10 centimeters and the diameter of its opening is 6 centimeters, how fast is the level of the liquid falling when the depth of the liquid is 5 centimeters?

5. An airplane, flying west at 400 miles per hour, goes over a certain town at 11:30 A.M., and a second plane at the same altitude, flying south at 500 miles per hour, goes over the town at noon. How fast are they separating at 1:00 P.M.? *Hint:* See Example 3.

6. A man on a dock is pulling in a rope fastened to the bow of a small boat. If the man's hands are 12 feet higher than the point where the rope is attached to the boat and if he is retrieving the rope at the rate of 3 feet per second, how fast is the boat approaching the dock when 20 feet of rope are still out?

7. A 20-foot ladder is leaning against a wall. If the bottom of the ladder is pulled along the level pavement directly away from the wall at 2 feet per second, how fast is the top of the ladder moving down the wall when the foot of the ladder is 4 feet from the wall?

8. Oil from a ruptured tanker spreads in a circular pattern. If the radius of the circle increases at the constant rate of 1.5 feet per second, how fast is the enclosed area increasing at the end of 2 hours?

9. Sand is pouring from a pipe at the rate of 16 cubic feet per second. If the falling sand forms a conical pile on the ground whose altitude is always $\frac{1}{4}$ the diameter of the base, how fast is the altitude increasing when the pile is 4 feet high? *Hint:* Refer to Figure 9 and use the fact that $V = \frac{1}{3}\pi r^2 h$.

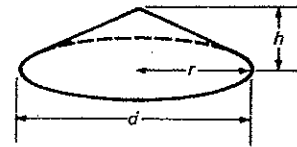


FIGURE 9

10. A child is flying a kite. If the kite is 90 feet above the child's hand level and the wind is blowing it on a horizontal course at 5 feet per second, how fast is the child paying out cord when 150 feet of cord is out? (Assume the cord forms a line—actually an unrealistic assumption.)

11. A swimming pool is 40 feet long, 20 feet wide, 8 feet deep at the deep end, and 3 feet deep at the shallow end; the bottom is rectangular (see Figure 10). If the pool is filled by pumping water into it at the rate of 40 cubic feet per minute, how fast is the water level rising when it is 3 feet deep at the deep end?

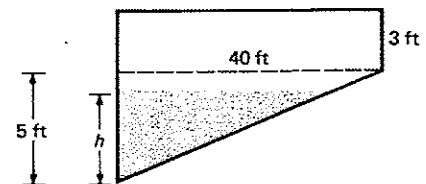


FIGURE 10

12. A particle P is moving along the graph of $y = \sqrt{x^2 - 4}$, $x \geq 2$, so that the x -coordinate of P is increasing at the rate of 5 units per second. How fast is the y -coordinate of P increasing when $x = 3$?

13. A metal disk expands during heating. If its radius increases at the rate of 0.02 inch per second, how fast is the area of one of its faces increasing when its radius is 8.1 inches?

14. Two ships sail from the same island port, one going north at 24 knots (24 nautical miles per hour) and the other east at 30 knots. The northbound ship departed at 9:00 A.M. and the eastbound ship left at 11:00 A.M. How fast is the distance between them increasing at 2:00 P.M.? *Hint:* Let $t = 0$ at 11:00 A.M.