

Related Rates - two rates; one affects the other

ex.1 If $A = \pi r^2$ find the rate of change of the area w.r.t. time.
(with respect to)

Differentiate both sides w.r.t time

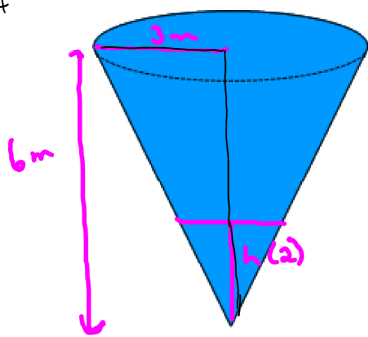
$$A = \pi r^2 \rightarrow A(t) = \pi (r(t))^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

rate of change of Area rate of change of radius

ex.2 A conical hopper is being filled with wheat at the rate of $0.5 \text{ m}^3/\text{s}$. The hopper is 6m high and 3m in radius at the top. Find the rate at which wheat is rising in the hopper when the depth is 2m.

$\frac{dV}{dt}$ rate of change of Volume $\frac{dh}{dt}$ rate of change of height of wheat



$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{h}{2}\right)^2 h$$

$$V = \frac{1}{3} \pi \frac{h^3}{4}$$

$$V = \frac{1}{12} \pi h^3$$

$$\frac{dV}{dt} = \frac{\pi}{12} (3h^2) \frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$$

$$0.5 = \frac{\pi}{4} (2)^2 \frac{dh}{dt}$$

$$\frac{1}{2\pi} = \frac{dh}{dt}$$

$\frac{1}{2\pi} \text{ m/s}$ when $h=2\text{m}$

$\frac{dV}{dt} = 0.5 \text{ m}^3/\text{s}$
 $\frac{dh}{dt} = ?$
when $h=2\text{m}$

*tricky!
similar triangles
used to sub one variable out

$$\frac{h}{6} = \frac{r}{3}$$

$$br = 3h$$

$$r = \frac{h}{2}$$

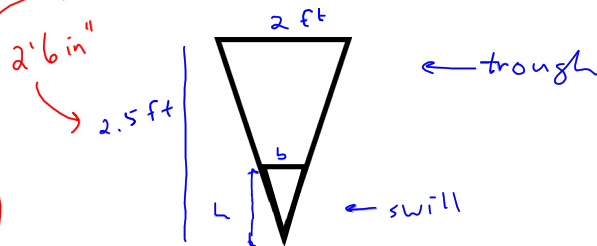
Need V and h in eqn because rates are $\frac{dV}{dt}$ and $\frac{dh}{dt}$ so sub "r" out

Sub in after take derivative and simplify

* can sub in any height and one rate of change to find other rate at different times.

Related Rates.notebook

Goes with
filling a
Trough
example
in reading



$$\frac{dV}{dt} \quad \frac{dh}{dt}$$

∴ need
V and h
in eqn

$$\frac{b}{2} = \frac{h}{2.5}$$

$$2.5b = 2h$$

$$0.8h = b$$

$$V = 5bh$$

$$V = 5(0.8h)h$$

$$V = 4h^2$$

$$\frac{dV}{dt} = 8h \frac{dh}{dt}$$

$$5 = 8(1.25) \frac{dh}{dt}$$

$$h = 1' 3''$$