

Rational Expressions and Equations: A4, A5, A6, R11

To find non-permissible values, equate denominator to 0

When simplifying:

- factor first (look for common factors, PSF, difference of squares)
- identify non-permissible values
- if multiplying, divide by common factors
- if dividing, flip (identify new non-permissible values), divide by common factors
- if adding/subtracting, need a common denominator

*Note: when solving you MUST check all possible answers

Reciprocal functions:

- graph the original function
- draw a vertical line at the x -intercepts, also known as the vertical asymptote(s)
- determine the invariant points ($y = 1$ and $y = -1$)
- graph the reciprocal graph
 - if the expression is linear (hyperbola)
 - if the expression is quadratic
 - funnel (1 vertical asymptote)
 - H-shape (2 vertical asymptotes)
 - Bump (No vertical asymptotes)

1. Identify non-permissible values: $\frac{x+1}{x^2-4x-5}$

$$x^2 - 4x - 5 = 0$$

$$(x-5)(x+1) = 0 \quad x \neq 5, -1$$

$$x = 5 \quad x = -1$$

2. Simplify: $\frac{2x^2-8x}{x-4}$

$$\frac{2x(x-4)}{\cancel{x-4}} = 2x, \quad x \neq 4$$

3. Simplify: $\frac{x^2-9}{2x+8} \div \frac{x-3}{x^2+5x+4}$ $x \neq -4, -1$ (Before Flip)

$$\frac{(x+3)\cancel{(x-3)}}{2\cancel{(x+4)}} \times \frac{\cancel{(x+4)}(x+1)}{\cancel{(x-3)}}$$

$$\frac{(x+3)(x+1)}{2}, x \neq -4, 3, -1$$

4. Simplify: $\frac{4x+3}{(x+1)(x-1)} + \frac{x}{x-1}$ LCD: $(x+1)(x-1)$

$$\frac{4x+3}{(x+1)(x-1)} + \frac{x}{x-1} \cdot \frac{(x+1)}{(x+1)}$$

$$\frac{4x+3}{(x+1)(x-1)} + \frac{x^2+x}{(x-1)(x+1)}$$

$$\frac{x^2+5x+3}{(x+1)(x-1)}, x \neq -1, 1$$

5. Simplify: $\frac{x^2-100}{144} \times \frac{36}{x+10}$

$$\frac{\cancel{(x+10)}(x-10)}{\cancel{144}4} \times \frac{\cancel{36}}{x+10}$$

$$\frac{x-10}{4}, x \neq -10$$

6. Simplify: $\frac{3x}{x+2} - \frac{x}{x-2}$ LCD: $(x+2)(x-2)$

$$\frac{3x}{x+2} \cdot \frac{x-2}{x-2} - \frac{x}{x-2} \cdot \frac{x+2}{x+2}$$

$$\frac{3x^2 - 6x}{(x+2)(x-2)} - \frac{x^2 + 2x}{(x-2)(x+2)}$$

$$\frac{3x^2 - 6x - x^2 - 2x}{(x+2)(x-2)} \Rightarrow \frac{2x^2 - 8x}{(x+2)(x-2)}, x \neq -2, 2$$

7. Solve: $\frac{3}{x} + \frac{4}{x^2} = 1$ $x \neq 0$

$$\frac{3}{x} \cdot \frac{x^2}{x^2} + \frac{4}{x^2} \cdot \frac{x^2}{x^2} = 1 \cdot \frac{x^2}{x^2}$$

$$3x + 4 = x^2$$

$$0 = x^2 - 3x - 4$$

$$0 = (x-4)(x+1)$$

$$x = 4 \quad x = -1$$

$$\boxed{x = -1, 4}$$

8. Solve: $\frac{6}{x-3} = \frac{x+3}{x^2-9} - 5$

$$\frac{6}{x-3} = \frac{x+3}{(x+3)(x-3)} - 5 \quad x \neq 3, -3$$

$$\frac{6}{\cancel{x-3}} \cdot \frac{(x+3)\cancel{(x-3)}}{\cancel{(x+3)}\cancel{(x-3)}} = \frac{x+3}{\cancel{(x+3)}\cancel{(x-3)}} - 5(x+3)\cancel{(x-3)}$$

$$6x + 18 = x + 3 - 5(x^2 - 9)$$

$$6x + 18 = x + 3 - 5x^2 + 45$$

$$5x^2 + 5x - 30 = 0$$

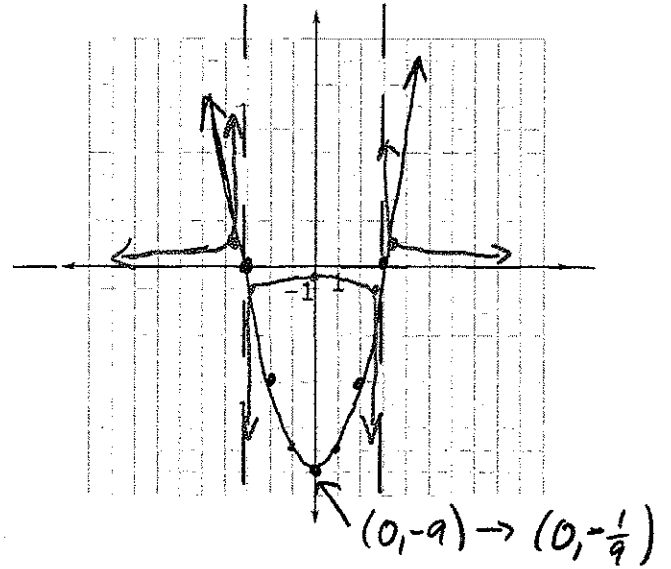
$$5(x^2 + x - 6) = 0$$

$$5(x+3)(x-2) = 0$$

$$\cancel{x = -3} \quad \boxed{x = 2}$$

9. Graph the function: $y = \frac{1}{x^2-9}$

$$y = \frac{1}{(x+3)(x-3)}$$



10. Graph the function: $y = \frac{1}{-2x-5}$

