Rates of Change and Tangent Lines

Average rate of change = $\frac{\text{amount of change}}{\text{time it takes}}$

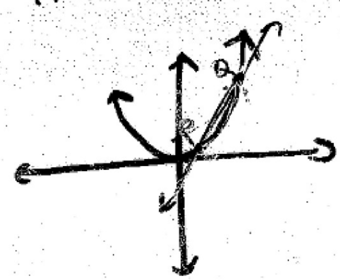
ex.! Find the average rate of change of $f(x) = x^4 - 3$ over the interval $[0, 2]$

avg rate of change $= \frac{f(2) - f(0)}{2-0}$

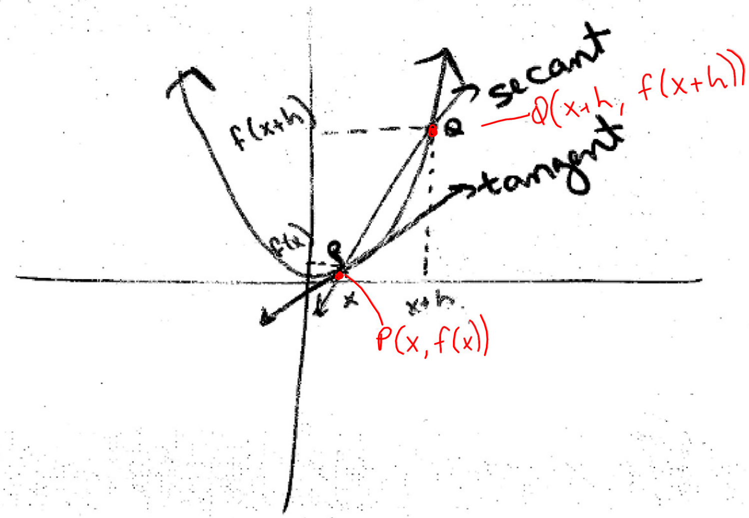
$= \frac{13 - (-3)}{2}$

$= 8$

A tangent to a curve at a point P on the curve is the limiting position of the secant line PQ as Q approaches P along the curve.



Average Rate of Change $= \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{slope} = \frac{f(x+h) - f(x)}{x+h-x}$$

∴ the slope of the tangent at P is

(h is the change in x-values) $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

<u>ex. 2</u> Find the slope of the tangent line at $x=2$ for the fcn. $f(x) = x^2$

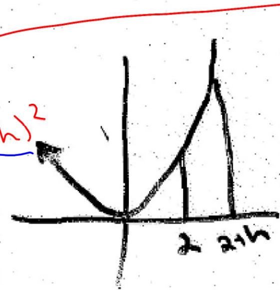
$$\text{secant slope} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{f(2+h) - f(2)}{h}$$

$$= \frac{(2+h)^2 - 2^2}{h}$$

$f(2+h) = (2+h)^2$

$f(2) = 2^2$



$(2+h)(2+h)$ FOIL

$4+2h+2h+h^2$

$$= \frac{\cancel{4}+4h+h^2-\cancel{4}}{h}$$

$$= \frac{\cancel{h}(4+h)}{\cancel{h}}$$

$$= 4+h$$

$\therefore$ tangent slope

$$\lim_{h\to 0} 4+h$$

$$4+0$$

$$4$$

eqn of tangent

$m=4$

$(2, 4)$

$x_1 \quad y_1$

$f(x)=x^2$

$f(2)=2^2$

$y-y_1=m(x-x_1)$ ← slope point form

$y-4=4(x-2)$

$y-4=4x-8$

$y-4=4x-8$

$0=4x-y-4$ ← general form

Slope of a Curve at a Point

The slope of the curve $y = f(x)$ at the point $P(a, f(a))$ is the number

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

provided the limit exists

The tangent line to the curve at P is the line through P w/ this slope

$y - y_1 = m(x - x_1)$

The normal line to the curve at a point is the line perpendicular to the tangent at that point.

↳ perpendicular lines have slopes that are negative reciprocals

ie 4 and $-\frac{1}{4}$

ex.2 Find the eqn of the tangent line to the curve $y = \frac{1}{2x}$ @ $(\frac{1}{2}, 1)$

$f(x) = \frac{1}{2x}$

$f(\frac{1}{2}+h) = \frac{1}{2(\frac{1}{2}+h)}$

$f(\frac{1}{2}) = \frac{1}{2(\frac{1}{2})}$

$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{f(\frac{1}{2}+h) - f(\frac{1}{2})}{h}$$

$$\lim_{h \to 0} \frac{\frac{1}{2(\frac{1}{2}+h)} - \frac{1}{2(\frac{1}{2})}}{h}$$

$$\lim_{h \to 0} \frac{\frac{1}{1+2h} - 1}{h}$$

$$\lim_{h \to 0} \frac{\frac{1 - (1+2h)}{1+2h}}{h}$$

$$\lim_{h\to 0} \frac{\frac{1-(1+2h)}{1+2h}}{h}$$

common denominator on top

$$\lim_{h\to 0} \frac{\frac{\cancel{1}-\cancel{1}-2h}{1+2h}}{h}$$

$$\lim_{h\to 0} \frac{-2}{1+2h} \div h$$

$$\cdot \frac{1}{h}$$

mult by reciprocal

$$\lim_{h\to 0} \frac{-2\cancel{h}}{1+2h} \cdot \frac{1}{\cancel{h}}$$

$$\lim_{h\to 0} \frac{-2}{1+2h}$$

$$\frac{-2}{1+2(0)}$$

$m = -2$

recall:

$f(x) = \frac{1}{2x}$

$f(\frac{1}{2}) = \frac{1}{2(\frac{1}{2})}$

$= 1$

$-2 \leftarrow$ slope

eqn of tangent

x_1, y_1

$(\frac{1}{2}, 1)$

$y - y_1 = m(x - x_1)$

$y - 1 = -2(x - \frac{1}{2})$

$y - 1 = -2x + 1$

$2x + y - 2 = 0$

eqn of normal line

use the negative reciprocal of the slope

$m = -2$

$\perp m = \frac{1}{2}$

$y - 1 = \frac{1}{2}(x - \frac{1}{2})$

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