

Radicals and Rationals

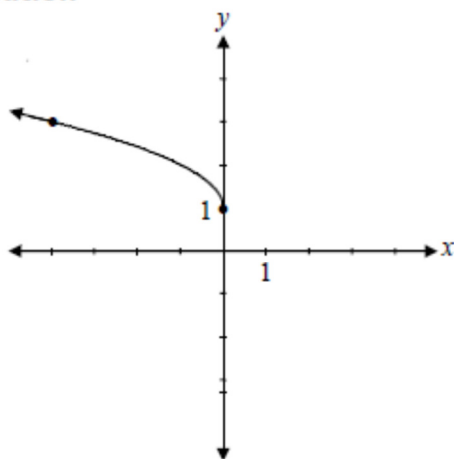
January 2014

Question 27

a) 3 marks b) 1 mark

a) Sketch the graph of the function $y = \sqrt{-x} + 1$.

Solution

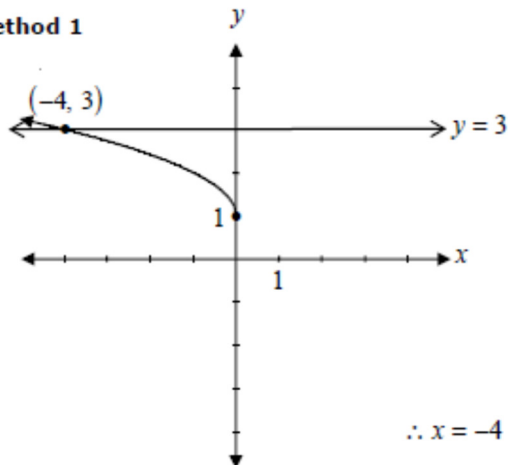


1 mark for general shape
1 mark for horizontal reflection
1 mark for vertical shift

3 marks

b) Determine the value of x when $y = 3$.

Method 1



$$\therefore x = -4$$

1 mark for consistent value of x

1 mark

Method 2

$$y = \sqrt{-x} + 1$$

$$3 = \sqrt{-x} + 1$$

$$2 = \sqrt{-x}$$

$$4 = -x$$

$$x = -4$$

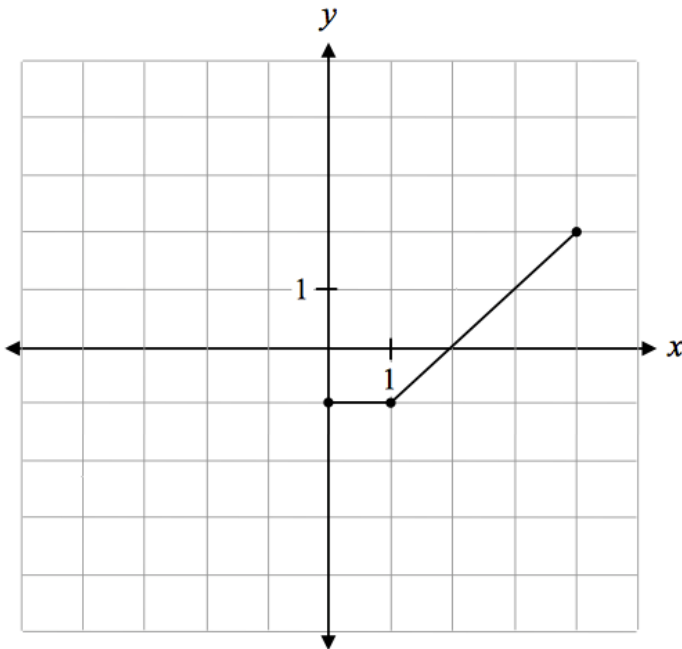
1 mark for consistent value of x

1 mark

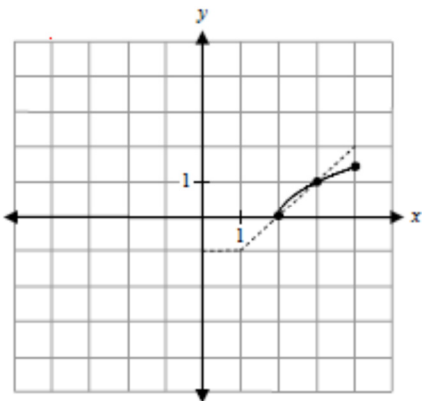
Question 34

2 marks

Given the graph of $y = f(x)$ below,



Sketch the graph of $y = \sqrt{f(x)}$.

Solution

1 mark for restricting the domain

$\frac{1}{2}$ mark for graph above $y = f(x)$ over the range $[0, 1]$

$\frac{1}{2}$ mark for graph below $y = f(x)$ over the range $[1, 2]$

2 marks

Question 30**1 mark**

Write the equation of the horizontal asymptote for the function $f(x) = \frac{x-3}{x-2}$.

Solution

$$y = 1$$

1 mark for equation of horizontal asymptote

1 mark**Question 36****2 marks**

Identify the domain and range of the following function:

$$f(x) = \frac{3}{x^2+1}$$

Solution

Domain: $\{x \in \mathbb{R}\}$

1 mark for domain

or

$(-\infty, \infty)$

Range: $\{y \in \mathbb{R} \mid 0 < y \leq 3\}$

1 mark for range

or

$(0, 3]$

2 marks

Question 44

4 marks

Sketch the graph of the following function:

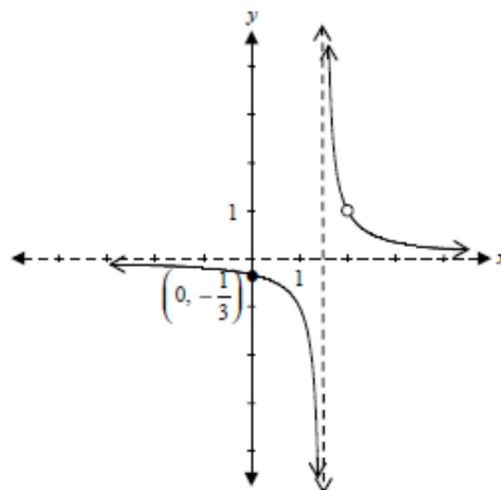
$$f(x) = \frac{x-2}{(2x-3)(x-2)}$$

Solution

$$\begin{aligned} f(x) &= \frac{x-2}{(2x-3)(x-2)} \\ &= \frac{1}{2x-3} \text{ with a point of discontinuity at } x=2 \end{aligned}$$

point of discontinuity: $f(2) = 1$ \therefore there is a point of discontinuity at $(2, 1)$

$$\begin{aligned} \text{y-intercept: } f(0) &= \frac{0-2}{(2(0)-3)(0-2)} \\ &= -\frac{2}{6} \\ &= -\frac{1}{3} \end{aligned}$$

1 mark for horizontal asymptote at $y = 0$ 1 mark for vertical asymptote at $x = \frac{3}{2}$ $\frac{1}{2}$ mark for graph left of vertical asymptote $\frac{1}{2}$ mark for graph right of vertical asymptote1 mark for point of discontinuity at $(2, 1)$; ($\frac{1}{2}$ mark for $x = 2$, $\frac{1}{2}$ mark for $y = 1$)

4 marks

June 2013**Question 34****2 marks**

The graph of a rational function, $f(x)$, has a point of discontinuity when $x = 2$ and an asymptote when $x = 4$. Write a possible equation for $f(x)$.

Solution

A possible equation is:

$$f(x) = \frac{x-2}{(x-2)(x-4)}$$

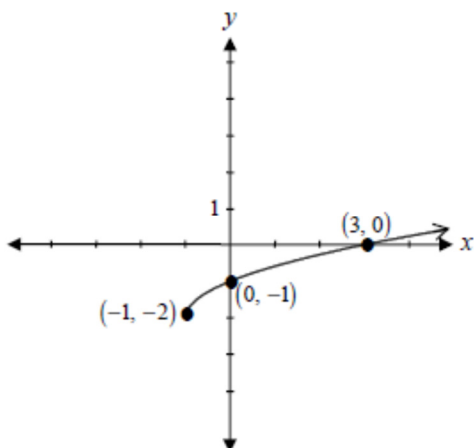
1 mark for $\frac{x-2}{x-2}$ (point of discontinuity when $x = 2$)

1 mark for $x-4$ in denominator (asymptote when $x = 4$)

2 marks

Question 37**3 marks**

Sketch the graph of $y = \sqrt{x+1} - 2$ and verify that the value of the x -intercept is the same as the solution to the equation $\sqrt{x+1} - 2 = 0$.

Solution

1 mark for general shape
 $\frac{1}{2}$ mark for horizontal shift
 $\frac{1}{2}$ mark for vertical shift

$$\begin{array}{l} \sqrt{x+1} = 2 \\ (\sqrt{x+1})^2 = (2)^2 \\ x+1 = 4 \\ x = 3 \end{array} \quad \text{or} \quad \begin{array}{l} \sqrt{x+1} - 2 = 0 \\ \sqrt{3+1} - 2 = 0 \\ \sqrt{4} - 2 = 0 \\ 0 = 0 \end{array}$$

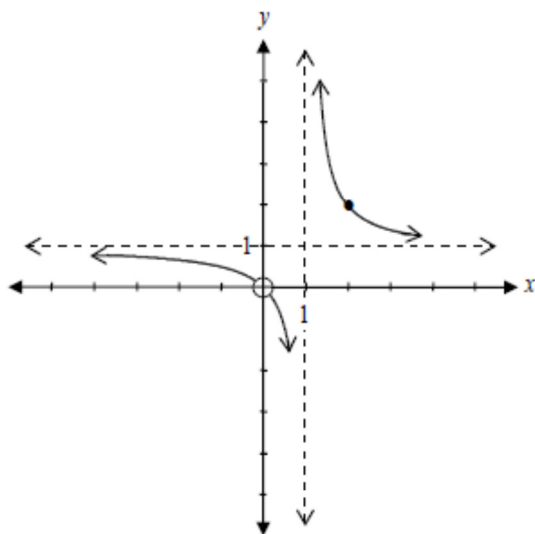
1 mark for verification

3 marks

Question 43

4 marks

Sketch the graph of the function $f(x) = \frac{x^2}{x^2 - x}$.

Solution

$$f(x) = \frac{x^2}{x(x-1)}$$

$$= \frac{x}{x-1} \text{ with a point of discontinuity where } x = 0$$

point of discontinuity: $f(0) = \frac{0}{0-1} = 0$

\therefore there is a point of discontinuity at $(0, 0)$.

divide:

$$\begin{array}{r} 1 \\ x-1 \overline{)x+0} \\ \underline{x-1} \\ 1 \end{array} \quad \text{or} \quad f(x) = \frac{x}{x-1}$$

$$= \frac{x-1+1}{x-1}$$

$$= 1 + \frac{1}{x-1}$$

$\therefore f(x) = \frac{1}{x-1} + 1$

\therefore horizontal asymptote at $y = 1$

\therefore vertical asymptote at $x = 1$

1 mark for vertical asymptote at $x = 1$

1 mark for horizontal asymptote at $y = 1$

1 mark for point of discontinuity at $(0, 0)$ or a point of discontinuity consistent with graph

$\frac{1}{2}$ mark for graph left of vertical asymptote

$\frac{1}{2}$ mark for graph right of vertical asymptote

4 marks