Lesson 5 Solving Radical Equations

A ***radical equation*** is an equation that contains at least one radical with a variable in the radicand. A solution to a radical equation is called the ***root*** of the equation.

↳ any value that makes an eqn true.

Steps to Solving Radical Equations

1. Isolate the radical with the variable in the radicand
2. Square both sides of the equation
3. Check your solutions or use restrictions to identify extraneous roots.

↳ will satisfy "new" eqn but may not satisfy the original eqn.

Examples
Solve each equation.

1. $(\sqrt{2x})^2 = 4^2$

Square both sides

$2x = 16$

$\boxed{x = 8}$

check:

$\sqrt{2(8)} = 4$

$\sqrt{16} = 4$

$4 = 4$ ✓

OR

use restrictions

$2x \geq 0$

$x \geq 0$

$8 \geq 0$ ✓

2. $(3\sqrt{x})^2 = 4^2$

square both sides

$9x = 16$

$x = \frac{16}{9}$

check

$3\sqrt{\frac{16}{9}} = 4$

$3\left(\frac{4}{3}\right) = 4$

$4 = 4$ ✓

3. $2\sqrt{x+1} - 7 = 13$

$2\sqrt{x+1} = 20$

$\sqrt{x+1} = 10$

square both sides $x+1 = 100$

$x = 99$

check

$2\sqrt{99+1} - 7 = 13$

$2(10) - 7 = 13$

$13 = 13$ ✓

4. $4\sqrt{x} + 3 = 5\sqrt{x} + 1$

combine like terms

$2 = \sqrt{x}$

$4 = x$

$x \geq 0$

$4 \geq 0$ ✓ or check

Example 2

Show that $\sqrt{2x-5} = \sqrt{x-7}$ has an extraneous root.

Use check

$\sqrt{2(-2)-5} = \sqrt{-2-7}$

$\sqrt{-9} = \sqrt{-9}$

$\emptyset$ we can't square root a -ve number

$2x - 5 = x - 7$

$x = -2$

"new" eqn

$x = -2$ satisfies the eqn $2x - 5 = x - 7$ but not the original eqn

$-2 \neq 7$

$\therefore$ the eqn has an extraneous root

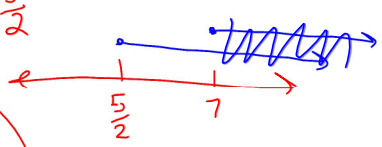
or restrictions

$2x - 5 \geq 0$ and $x - 7 \geq 0$

$2x \geq 5$

$x \geq \frac{5}{2}$

$x \geq 7$



$\therefore x \geq 7$

Example 2

Solve.

1. $\sqrt{x+3}+5=0$

$\sqrt{x+3} = -5$

ϕ ← we would never get a -ve value for a principal square root

or

$x+3 = 25$

$x = 22$ rej

ϕ

check $\sqrt{22+3} + 5 \neq 0$

$5 + 5 \neq 0$

$10 \neq 0$

2. $\sqrt{2x+7} - x = -4$

$(\sqrt{2x+7})^2 = (x-4)^2$

$2x+7 = x^2 - 8x + 16$

$0 = x^2 - 10x + 9$

$0 = (x-1)(x-9)$

$x-1=0 \quad x-9=0$

$x = 1$ reject $\quad x = 9$

P 9
S -10
F -9, -1

$(x-4)(x-4)$

$x^2 - 4x - 4x + 16$

check

$x=1$

$\sqrt{2(1)+7} - 1 \neq -4$

$\sqrt{9} - 1 \neq -4$

$2 \neq -4$

$x = 9$

$\sqrt{2(9)+7} - 9 = -4$

$\sqrt{25} - 9 = -4$

$5 - 9 = -4$

$-4 = -4$ ✓

3. $\sqrt{2x+3} - \sqrt{x+2} = 2$

$\left(\sqrt{2x+3}\right)^2 = \left(2 + \sqrt{x+2}\right)^2$

$2x+3 = 4 + 4\sqrt{x+2} + x+2$

$(x-3)^2 = \left(4\sqrt{x+2}\right)^2$ square again

$x^2 - 6x + 9 = 16(x+2)$

$x^2 - 6x + 9 = 16x + 32$

$x^2 - 22x - 23 = 0$

$(x-23)(x+1) = 0$

$x = 23$ $x \neq -1$

check

$\sqrt{2(23)+3} = 2 + \sqrt{23+2}$

$7 = 2 + 5$

$7 = 7$ ✓

$\sqrt{2(-1)+3} = 2 + \sqrt{-1+2}$

$1 \neq 3$

#3a, c, e, g, i, k