Lesson 4 Dividing Radicals

Rationalizing the Denominator (Monomials)

The process of changing the denominator from a radical (irrational number) to a rational number is called rationalizing the denominator. This does not change the value of the expression, only the form of the expression since when a fraction is multiplied by 1, its value does not change.

For example, $\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ $\frac{\sqrt{3}}{\sqrt{3}} = 1$ so not changing the value

The denominator is now a rational number and $\frac{2\sqrt{3}}{3}$ is considered in simplest form.

Examples

Express in simplest form.

1. $\frac{5}{\sqrt{2}}$ $\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$\frac{5\sqrt{2}}{2}$

2. $\frac{4}{2\sqrt{7}}$ $\frac{2}{\sqrt{7}}\left(\frac{\sqrt{7}}{\sqrt{7}}\right)$

$\frac{2\sqrt{7}}{7}$

3. $\frac{\sqrt{12}}{\sqrt{x}}$ $\left(\frac{\sqrt{x}}{\sqrt{x}}\right)$

$\frac{\sqrt{12x}}{x}$ $x > 0$

$\frac{2\sqrt{3x}}{x}$ simplify all complex radicals

4. $\frac{5\sqrt{7}+3}{\sqrt{7}}$ $\left(\frac{\sqrt{7}}{\sqrt{7}}\right)$

$\frac{5(7)+3\sqrt{7}}{7}$

$\frac{35+3\sqrt{7}}{7}$

5. $\frac{6\sqrt{2}-4\sqrt{3}}{\sqrt{18}}$

$\frac{(6\sqrt{2}-4\sqrt{3})}{3\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$\frac{6(2)-4\sqrt{6}}{3(2)}$

$\frac{12-4\sqrt{6}}{6}$ → GCF $\frac{4(3-\sqrt{6})}{6}$ reduce $\frac{4}{6}$ to $\frac{2}{3}$ $\frac{2(3-\sqrt{6})}{3}$

divide each term by 2

$\boxed{\frac{6-2\sqrt{6}}{3}}$

Rationalizing the Denominator (Binomials)

To rationalize a denominator containing a binomial we use a difference of squares. The expressions $(a + b)$ and $(a - b)$ are conjugates and have a product that is a difference of squares.

$\sqrt{3} + \sqrt{2}$ has a conjugate of $\sqrt{3} - \sqrt{2}$

$2\sqrt{3} - 3\sqrt{5}$ has a conjugate of $2\sqrt{3} + 3\sqrt{5}$

Examples

Simplify the following radicals

1. $\frac{2}{\sqrt{5}-\sqrt{2}}$ $(\sqrt{5}+\sqrt{2})$ multiply by the conjugate $\sqrt{5}+\sqrt{2}$

$\frac{2\sqrt{5}+2\sqrt{2}}{5-2}$

$\frac{2\sqrt{5}+2\sqrt{2}}{3}$

$(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

$5 + \sqrt{10} - \sqrt{10} - 2$

inside terms always add to 0 when multiplying conjugates

2. $\frac{\sqrt{2}+1}{\sqrt{2}-1}$ $\left(\frac{\sqrt{2}+1}{\sqrt{2}+1}\right)$

$\frac{2+\sqrt{2}+\sqrt{2}+1}{2-1}$

$\frac{3+2\sqrt{2}}{1}$

$3+2\sqrt{2}$

#3 a, c, e, g, i, m

8 a, c, g, j

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